Day 296 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 06:20 (London time).
It takes 9 hours 45 minutes.
When it is 12:00 in London it is 20:00 in Tokyo.
At what time, in Tokyo time, does the flight land? Say if it is on a different day.

Arrival in London time = 06:20 + 9 h 45 min = 16:05
Tokyo is 8 hours ahead of London, so add 8 hours
00:05 the next day
Answer: 00:05 the next day

2 Arithmetic sequences

The 5th term of an arithmetic sequence is 31.
The 8th term of the sequence is 43.
(a) Find an expression for the nth term of the sequence.
(b) Is 119 a term of the sequence? You must show your working.

(a) Common difference = (43 − 31) ÷ (8 − 5) = 4
nth term = 4n + c. For n = 5: 4 × 5 + c = 31, so c = 11
nth term = 4n + 11
(b) Solve 4n + 11 = 119: 4n = 108, so n = 27
n is a positive whole number, so 119 is the 27th term
Answer: (a) 4n + 11 (b) Yes, it is the 27th term

3 Percentages

Q is 40% less than A.
A is 15% more than C.
Work out Q as a percentage of C.

Q = 0.6 × A and A = 1.15 × C
Q = 0.6 × 1.15 × C = 0.69 × C
0.69 × 100 = 69%
Answer: 69%

4 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 63°} = \frac{15}{\sin 35°}$

$x = \frac{15 \times \sin 63°}{\sin 35°} = 23.301$

x = 23.3 cm (1 d.p.)
Answer: AC = 23.3 cm

5 Fractions removed from a ratio

The ratio of blue to green sweets in a tin is 4 : 5.
Then $\frac{2}{3}$ of the blue and $\frac{1}{2}$ of the green are taken out.
Work out the ratio of blue to green now.

Left: $\frac{1}{3} \times 4 = \frac{4}{3}$ of the blue part, $\frac{1}{2} \times 5 = \frac{5}{2}$ of the green part

$\frac{4}{3} : \frac{5}{2}$

Multiply by 6: 8 : 15
Answer: 8 : 15

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6 Iteration: rearranging equations

(a) Show that the equation $x^3 - 4x - 6 = 0$ can be rearranged to give $x = \sqrt[3]{4x + 6}$ (Calculator)

(b) Use the iteration formula $x_{n+1} = \sqrt[3]{4x_n + 6}$ with $x_0 = 2$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^3 - 4x - 6 = 0$: $x^3 = 4x + 6$

Take the cube root of both sides: $x = \sqrt[3]{4x + 6}$

(b) $x_1 = \sqrt[3]{4 \times 2 + 6} = 2.41014...$

$x_2 = \sqrt[3]{4 \times x_1 + 6} = 2.50083...$

$x_1 = 2.410$ and $x_2 = 2.501$ (3 d.p.)

Answer: (b) $x_1 = 2.410$, $x_2 = 2.501$

7 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.

(a) Work out the interquartile range for each.

(b) Compare the distributions of A and B.

(a) IQR of A = 80 − 65 = 15

IQR of B = 55 − 35 = 20

(b) Class A has the higher median (70 compared with 40), so on average Class A scored higher

Class A has the smaller interquartile range (15 compared with 20), so its marks are more consistent

Answer: (a) A: 15, B: 20 (b) A has the higher median, A has the smaller IQR

8 Radius from chord and height of arc

A circular arc has a chord AB of length 24 cm.

The greatest distance between the chord and the arc is 4 cm.

Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 4

AM = 24 ÷ 2 = 12. Pythagoras in triangle OMA: $r^2 = (r - 4)^2 + 12^2$

$r^2 = r^2 - 8r + 16 + 144$

8r = 160, so r = 20 cm

Answer: 20 cm

9 Proof with indices

Simplify fully $\frac{7^{k+2} - 7^k}{48}$

$7^{k+2} - 7^k = 7^k(7^2 - 1) = 7^k \times 48$

Divide by 48: $\frac{48 \times 7^k}{48} = 7^k$

Answer: 7^k

10 Algebraic proof

Prove that the product of (6n − 1) and (6n + 1) is always 1 less than a multiple of 36.

$(6n - 1)(6n + 1) = 36n^2 + 6n - 6n - 1$

$= 36n^2 - 1$

$36n^2$ is a multiple of 36, so the product is 1 less than a multiple of 36

Answer: Proved