Day 295 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x + 30)°$, $(x - 5)°$, $(x + 25)°$, $(x + 15)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$5x + 65 = 540$, so $5x = 475$
$x = 95$
(b) The angles are 95°, 125°, 90°, 120°, 110°
The largest is 125°
Answer: (a) x = 95 (b) 125°

2 Weighted percentages

The ratio of juniors to seniors in a school is 3 : 2.
15% of the juniors and 60% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 60 juniors and 40 seniors
15% of 60 = 9 and 60% of 40 = 24
Together: 33 out of 100
33 ÷ 100 × 100 = 33%
Answer: 33%

3 Inequalities

(a) Solve $-3 - 2x \geq 17$
(b) n is an integer. Write down all the values of n that satisfy
$-4 \leq 2n < 8$

(a) Add 3: $-2x \geq 20$
Divide by -2 and reverse the inequality sign: $x \leq -10$
(b) Subtract 0: $-4 \leq 2n < 8$
Divide by 2: $-2 \leq n < 4$
Integers: $-2, -1, 0, 1, 2, 3$
Answer: (a) $x \leq -10$ (b) $-2, -1, 0, 1, 2, 3$

4 Rational and irrational numbers

Here are six numbers.
$3 + \sqrt{12}$, $\frac{\sqrt{18}}{\sqrt{2}}$, $\sqrt{8} \times \sqrt{2}$, $\sqrt{\frac{9}{16}}$, $2\pi - \pi$, $(\sqrt{6} + 2)(\sqrt{6} - 2)$
Write down the irrational numbers.

Simplify each number:
$3 + \sqrt{12} = 3 + 2\sqrt{3}$; $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$; $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$
$\sqrt{\frac{9}{16}} = \frac{3}{4}$; $2\pi - \pi = \pi$; $(\sqrt{6} + 2)(\sqrt{6} - 2) = 6 - 4 = 2$
Answer: $3 + \sqrt{12}$, $2\pi - \pi$

5 Fractions removed from a ratio

The ratio of men to women people at a meeting is 5 : 3.
Then $\frac{3}{4}$ of the men and $\frac{1}{4}$ of the women are taken out.
Work out the ratio of men to women now.

Left: $\frac{1}{4} \times 5 = \frac{5}{4}$ of the men part, $\frac{3}{4} \times 3 = \frac{9}{4}$ of the women part
$\frac{5}{4} : \frac{9}{4}$
Multiply by 4: 5 : 9
Answer: 5 : 9

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6 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 2^2 + 10^2 = 104$
$AG^2 = AC^2 + CG^2 = 104 + 11^2 = 225$
AG = 15 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{11}{\sqrt{104}}$

angle GAC = 47.2° (1 d.p.)
Answer: (a) 15 cm (b) 47.2°

7 Equations in disguise

Solve $x - 7\sqrt{x} + 10 = 0$

Let $u = \sqrt{x}$: $u^2 - 7u + 10 = 0$
Factorise: $(u - 5)(u - 2) = 0$, so u = 5 or u = 2
$\sqrt{x} = 5$ or $\sqrt{x} = 2$
x = 25 or x = 4
Answer: x = 4 or x = 25

8 Percentage change in area and volume

The radius of a sphere is decreased by 60%.
Work out the percentage change in the volume of the sphere.

Radius multiplier = 0.4
Volume multiplier = $0.4^3 = 0.064$
$0.064 - 1 = -0.936$, so a 93.6% decrease
Answer: 93.6% decrease

9 Bearings with the sine and cosine rules

A walker leaves a camp P and travels 74 km on a bearing of 140° to Q. (Calculator)
It then travels 46 km on a bearing of 070° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 140° + 180° = 320°
Angle PQR = 110° (the difference between the bearings 320° and 070°)
$PR^2 = 74^2 + 46^2 - 2 \times 74 \times 46 \times \cos 110° = 9920.5$, so PR = 99.6 km

(b) $\cos(QPR) = \frac{74^2 + PR^2 - 46^2}{2 \times 74 \times PR}$, so angle QPR = 25.7°

R is anticlockwise of the direction PQ, so the bearing is 140° − 26° = 114°
Answer: (a) 99.6 km (b) 114°

10 Counting factors

How many factors does 3430 have?
Include 1 and 3430.

$3430 = 2 \times 5 \times 7^3$
Add 1 to each power and multiply: $2 \times 2 \times 4 = 16$
Answer: 16