Day 294 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 18:30 (London time).
It takes 7 hours 10 minutes.
When it is 12:00 in London it is 14:00 in Cairo.
At what time, in Cairo time, does the flight land? Say if it is on a different day.

Arrival in London time = 18:30 + 7 h 10 min = 01:40 the next day
Cairo is 2 hours ahead of London, so add 2 hours
03:40 the next day
Answer: 03:40 the next day

2 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 25^2 - 7^2 = 576$
AM = 24 cm
AB = 2 × 24 = 48 cm
(b) sin(angle AOM) = $\frac{24}{25}$
Angle AOM = 73.74°, so angle AOB = 2 × 73.74 = 147.5°
Answer: (a) 48 cm (b) 147.5°

3 HCF and LCM

(a) Write 784 and 700 as products of prime factors.
(b) Find the HCF and the LCM of 784 and 700.
Give your answers as numbers.

(a) $784 = 2^4 \times 7^2$ and $700 = 2^2 \times 5^2 \times 7$
(b) HCF = $2^2 \times 7 = 28$
LCM = $2^4 \times 5^2 \times 7^2 = 19{,}600$
Answer: (a) $2^4 \times 7^2$; $2^2 \times 5^2 \times 7$ (b) HCF = 28, LCM = 19,600

4 Average speed

Maya drives 60 km at an average speed of 40 km/h.
The drive then continues for another 30 km at an average speed of 60 km/h.
Work out the average speed for the whole journey.

Time for first part = 60 ÷ 40 = 1.5 hours
Time for second part = 30 ÷ 60 = 0.5 hours
Total distance = 90 km, total time = 2 hours
Average speed = 90 ÷ 2 = 45 km/h
Answer: 45 km/h

5 Area and quadratic equations

The diagram shows a rectangle of length (3x + 2) cm, width (2x + 3) cm and area 154 cm^2.
(a) Show that $6x^2 + 13x - 148 = 0$.
(b) Work out the perimeter of the rectangle.

(a) (3x + 2)(2x + 3) = 154
Expand: $6x^2 + 13x + 6 = 154$, so $6x^2 + 13x - 148 = 0$
(b) Factorise: (x − 4)(6x + 37) = 0
x = 4 (x = $-\frac{37}{6}$ is not possible as a length would be negative)
Length = 14 cm and width = 11 cm
Perimeter = 2 × (14 + 11) = 50 cm
Answer: (b) 50 cm

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6 Probabilities with an unknown

A biased four-sided dice lands on one of four outcomes with these probabilities.
1: x; 2: 3x + 0.2; 3: 2x + 0.1; 4: 0.1
(a) Work out the value of x.
(b) The biased four-sided dice is used 120 times. Work out an estimate for the number of times it lands on 1.

(a) Probabilities add up to 1: x + 3x + 0.2 + 2x + 0.1 + 0.1 = 1
6x + 0.4 = 1, so 6x = 0.6
x = 0.1
(b) P(1) = x = 0.1, so 120 × 0.1 = 12
Answer: (a) x = 0.1 (b) 12

7 Equation of a circle

A and B are the points (−2, 1) and (−8, 11).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{-2 + (-8)}{2}, \frac{1 + 11}{2}) = (-5, 6)$
$r^2 = (-2 - (-5))^2 + (1 - 6)^2 = 9 + 25 = 34$
$(x + 5)^2 + (y - 6)^2 = 34$
Answer: $(x + 5)^2 + (y - 6)^2 = 34$

8 Terms of a quadratic sequence

The nth term of a sequence is $2n^2 + 3$.
(a) Work out the 7th term.
(b) Is 397 a term of the sequence? You must show your working.

(a) $2 \times 7^2 + 0 \times 7 + 3 = 101$
(b) Solve $2n^2 + 3 = 397$, so $2n^2 - 394 = 0$
$b^2 - 4ac = 0^2 - 4 \times 2 \times (-394) = 3152$, which is not a square number
So n is not a whole number, and 397 is not a term
Answer: (a) 101 (b) No

9 Volume and surface area of composite solids

A solid is made from a cylinder of height 5 cm and a hemisphere. Both have radius 3 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cylinder: $\pi \times 3^2 \times 5 = 45\pi$
Hemisphere: $\frac{1}{2} \times \frac{4}{3} \times \pi \times 3^3 = 18\pi$
Total = $45\pi + 18\pi = 63\pi$ cm^3
Answer: 63π cm^3

10 Probability and quadratic equations

A bag contains some counters. 6 of them are green and the rest are white.
Two counters are taken at random without replacement.

The probability that both are green is $\frac{5}{12}$.

Work out how many counters are white.

Let there be n counters altogether
P(both green) = $\frac{6}{n} \times \frac{5}{n-1} = \frac{30}{n(n-1)}$
$\frac{30}{n(n-1)} = \frac{5}{12}$, so $n(n - 1) = 72$, giving $n^2 - n - 72 = 0$
$(n - 9)(n + 8) = 0$, so n = 9 (n cannot be negative)
Number of counters that are white = 9 − 6 = 3
Answer: 3