Day 293 **S Tier 10-a-day: Model Solutions**

1 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 5 cm and inner radius 4.5 cm. (Calculator)
The density of the metal is 11.3 g/cm^3.
The mass of the pipe is 12.6 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 12600 g, so volume = mass ÷ density = 12600 ÷ 11.3 = 1115.0 cm^3
Cross-sectional area = $\pi \times 5^2 - \pi \times 4.5^2 = 14.923$ cm^2
Length = volume ÷ area = 1115.0 ÷ 14.923 = 74.7 cm (3 s.f.)
Answer: 74.7 cm

2 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 24 is taken. Work out how many come from Group D.
(b) A different stratified sample has 6 from Group C. Work out the size of this sample.

Total = 300 + 150 + 150 + 600 = 1200

(a) Group D: $\frac{600}{1200} \times 24 = 12$

(b) Fraction sampled = 6 ÷ 150 = $\frac{1}{25}$, so sample size = $\frac{1}{25} \times 1200 = 48$

Answer: (a) 12 (b) 48

3 Time zones

A flight leaves London at 14:30 (London time).
It takes 8 hours 55 minutes.
When it is 12:00 in London it is 14:00 in Cairo.
At what time, in Cairo time, does the flight land? Say if it is on a different day.

Arrival in London time = 14:30 + 8 h 55 min = 23:25
Cairo is 2 hours ahead of London, so add 2 hours
01:25 the next day
Answer: 01:25 the next day

4 Mean of a changing set

The mean of 9 numbers is 6.
A new number is added and the mean of all 10 numbers is 11.
Work out the new number.

Total of the 9 numbers = 9 × 6 = 54
New total = 10 × 11 = 110
New number = 110 − 54 = 56
Answer: 56

5 Pythagoras with algebra

ABC is a right-angled triangle with sides (x + 2) cm, (2x − 1) cm and (x + 3) cm. The longest side is (x + 3) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 2)^2 + (2x - 1)^2 = (x + 3)^2$
Expand and collect terms: $4x^2 - 6x - 4 = 0$
Factorise: $2(x - 2)(2x + 1) = 0$
$x = 2$ or $x = -\frac{1}{2}$, and x must be positive, so $x = 2$

(b) The two shorter sides are 4 cm and 3 cm

Area = $\frac{1}{2} \times 4 \times 3 = 6$ cm^2

Answer: (a) x = 2 (b) 6 cm^2

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6 Forming and solving quadratic equations

A rectangle has length (x + 6) cm and width (x + 4) cm.
The area of the rectangle is 195 cm^2.
(a) Show that $x^2 + 10x - 171 = 0$
(b) Work out the perimeter of the rectangle.

(a) $(x + 6)(x + 4) = 195$, so $x^2 + 10x + 24 = 195$
$x^2 + 10x - 171 = 0$
(b) $x^2 + 10x - 171 = 0$: $(x - 9)(x + 19) = 0$
$x = 9$ ($x = -19$ is not possible as lengths are positive)
Length = 15 cm and width = 13 cm, so perimeter = 2 × (15 + 13) = 56 cm
Answer: (b) 56 cm

7 Combined transformations

The point P(−4, 3) is rotated 90° clockwise about the origin.
The image is then reflected in the line $y = -x$.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (3, 4)
After the second transformation: P'' = (−4, −3)
(b) Follow a general point (x, y) through both transformations: (x, y) → (y, −x) → (x, −y)
The single mapping is (x, y) → (x, −y)
This is a reflection in the x-axis
Answer: (a) (−4, −3) (b) Reflection in the x-axis

8 Fraction problems

A bag contains marbles.
$\frac{1}{4}$ of the marbles are yellow and $\frac{3}{5}$ of the marbles are black.
The rest of the marbles, 9 of them, are white.
How many marbles are in the bag?

Fraction that are white = $1 - \frac{1}{4} - \frac{3}{5} = \frac{3}{20}$

$\frac{3}{20}$ of the total is 9

Total = $9 \div \frac{3}{20} = 60$

Answer: 60

9 Successive percentage change

The value of a car is decreased by 50%.
The new amount is then decreased by 25%.
Work out the overall percentage change.

Multipliers: 0.5 and 0.75
0.5 × 0.75 = 0.375
0.375 is a decrease of 62.5%
Answer: 62.5% decrease

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10 Equations with algebraic fractions

Solve $\frac{2}{x+2} - \frac{5}{x+5} = \frac{3}{4}$

Multiply every term by $(x + 2)(x + 5)$: $2(x + 5) - 5(x + 2) = \frac{3}{4} \times (x + 2)(x + 5)$

Multiply by 4 to clear the fraction: $8(x + 5) - 20(x + 2) = 3(x + 2)(x + 5)$

Expand and rearrange so that one side is 0: $x^2 + 11x + 10 = 0$

Factorise: $(x + 1)(x + 10) = 0$

$x = -10$ or $x = -1$

Answer: $x = -10$ or $x = -1$