Day 292 **S Tier 10-a-day: Model Solutions**

1 Surds

Given that $\sqrt{63} + \sqrt{175} = \sqrt{n}$,
find the value of n.

$\sqrt{63} = 3\sqrt{7}$ and $\sqrt{175} = 5\sqrt{7}$
$3\sqrt{7} + 5\sqrt{7} = 8\sqrt{7}$
$8\sqrt{7} = \sqrt{448}$, so n = 448
Answer: n = 448

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(3x - 15)°$
Angle PQR = $(2x)°$
Angle PRS = $(4x + 4)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$4x + 4 = 5x - 15$
$x = 19$
(b) Angle PRS = 80° so angle PRQ = 180° − 80° = 100°
Answer: (a) x = 19 (b) 100°

3 Solving quadratics by factorising

Solve $8x^2 - 6x = -1$

Rearrange so that one side is 0: $8x^2 - 6x + 1 = 0$
Factorise: $(4x - 1)(2x - 1) = 0$
$4x - 1 = 0$ or $2x - 1 = 0$
$x = \frac{1}{4}$ or $x = \frac{1}{2}$
Answer: x = $\frac{1}{4}$ or x = $\frac{1}{2}$

4 Percentage change

A 30% decrease followed by a 60% increase is equivalent to a single percentage change.
Work out the single percentage change that has the same effect.

Multipliers: 0.7 and 1.6
Overall multiplier = 0.7 × 1.6 = 1.12
1.12 − 1 = 0.12, so an overall increase of 12%
Answer: 12% increase

5 Distance-time graphs

The graph shows Zane's cycle ride to a café and back.
(a) Work out Zane's speed on the way back.
(b) Work out Zane's average speed for the whole ride, including the stop.

(a) Return: 20 km in 2.5 hours, so speed = 20 ÷ 2.5 = 8 km/h
(b) Total distance = 2 × 20 = 40 km, total time = 5 hours
Average speed = 40 ÷ 5 = 8 km/h
Answer: (a) 8 km/h (b) 8 km/h

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6 Combined transformations

The point P(−4, 2) is reflected in the x-axis.
The image is then reflected in the y-axis.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−4, −2)
After the second transformation: P'' = (4, −2)
(b) Follow a general point (x, y) through both transformations: (x, y) → (x, −y) → (−x, −y)
The single mapping is (x, y) → (−x, −y)
This is a rotation 180° about the origin
Answer: (a) (4, −2) (b) Rotation 180° about the origin

7 Powers of standard form

Work out the square root of 5.29×10^{-8}.
Give your answer in standard form.

$5.29 = 2.3 \times 2.3$
$\sqrt{(10^{-8})} = 10^{-4}$
$\sqrt{(5.29 \times 10^{-8})} = 2.3 \times 10^{-4}$
Answer: 2.3×10^{-4}

8 Perpendicular bisector

A is the point (−1, 1) and B is the point (−5, −7).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form y = mx + c.

Midpoint of AB = $(\frac{-1 + (-5)}{2}, \frac{1 + (-7)}{2}) = (-3, -3)$

Gradient of AB = $\frac{-7 - 1}{-5 - (-1)} = 2$

The perpendicular gradient is $-1 \div 2 = -\frac{1}{2}$

Using the midpoint: $y + 3 = -\frac{1}{2}(x + 3)$

$y = -\frac{1}{2}x - \frac{9}{2}$

Answer: $y = -\frac{1}{2}x - \frac{9}{2}$

9 Histograms

The histogram shows the times of people spent waiting at a clinic.
(a) Work out the total number of people.
(b) Estimate the number of people with a time between 20 minutes and 55 minutes.

Frequency = frequency density × class width
Multiply each height by its class width: 10, 16, 16, 8, 8
(a) Total = 58
(b) 20 to 30: 0.8 × 10 = 8
Whole bar between: 16
40 to 55: 0.4 × 15 = 6, so estimate = 30
Answer: (a) 58 (b) 30

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10 Circle and line intersections

Solve the simultaneous equations

$x^2 + y^2 = 13$

$y = x + 5$

Substitute into the first equation: $x^2 + (x + 5)^2 = 13$

Expand: $2x^2 + 10x + 12 = 0$

Divide by 2: $x^2 + 5x + 6 = 0$

Factorise: $(x + 3)(x + 2) = 0$, so $x = -3$ or $x = -2$

$y = x + 5$ gives $y = 2$ and $y = 3$

Answer: $x = -3$, $y = 2$ and $x = -2$, $y = 3$