**Day 291** **S Tier 10-a-day: Model Solutions**

## 1 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 58°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: 360 − 90 − 90 − 58 = 122°
(b) The angle at the circumference is half the angle at the centre: 122 ÷ 2 = 61°
**Answer: (a) 122° (b) 61°**

## 2 Capture and recapture

50 fish in a lake are caught, tagged and put back.
Later, a second sample of 120 fish is caught. 8 of these are tagged.
(a) Work out an estimate for the number of fish in a lake.
(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 8 out of 120
This should match the proportion tagged in the whole population: $\frac{8}{120} = \frac{50}{N}$
N = 50 × 120 ÷ 8 = 750
(b) No fish are born, die or leave between the samples, and the fish mix evenly
**Answer: (a) 750 (b) The population stays the same between samples**

## 3 Comparing costs

FitLife has a joining fee of £2.50 plus £4.50 per month.
GymNow has a joining fee of £35.50 plus £1.50 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 19 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when 2.50 + 4.50n = 35.50 + 1.50n
3n = 33, so n = 11
(b) FitLife: £2.50 + £4.50 × 19 = £88
GymNow: £35.50 + £1.50 × 19 = £64
GymNow is cheaper by £24
**Answer: (a) 11 (b) GymNow, by £24**

## 4 Exchange rates

Leila changes £500 into euros at £1 = €1.2.
Leila spends €100 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Leila get back?

£500 × 1.2 = €600
600 − 100 = €500
500 ÷ 1.6 = £312.50
**Answer: £312.50**

## 5 Area of a segment

The circle has centre O and radius 11 cm. A and B are points on the circle and angle AOB = 60°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{60}{360} \times \pi \times 11^2 = 63.355$ cm$^2$
Area of triangle OAB = $\frac{1}{2} \times 11 \times 11 \times \sin 60° = 52.395$ cm$^2$
Segment = sector − triangle = 63.355 − 52.395 = 10.961
= 11.0 cm$^2$ (3 s.f.)
**Answer: 11.0 cm$^2$**

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## 6 Forming and solving quadratic equations

The product of two positive numbers is 48.
The larger number is 2 more than the smaller number.
Let the smaller number be n.
(a) Show that $n^2 + 2n - 48 = 0$
(b) Work out the two numbers.

(a) The numbers are n and n + 2, so $n(n + 2) = 48$
$n^2 + 2n = 48$, so $n^2 + 2n - 48 = 0$
(b) Factorise: $(n - 6)(n + 8) = 0$
$n = 6$ or $n = -8$. n is positive, so $n = 6$
The numbers are 6 and 8
**Answer: (b) 6 and 8**

## 7 Exact trigonometric ratios from a given ratio

x is an acute angle and $\cos x = \frac{2}{11}$.
(a) Find the exact value of sin x.
(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\sin^2 x + \cos^2 x = 1$, so $\sin^2 x = 1 - (\frac{2}{11})^2 = \frac{117}{121}$

x is acute, so sin x is positive: $\sin x = \frac{3\sqrt{13}}{11}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{3\sqrt{13}}{11} \div \frac{2}{11} = \frac{3\sqrt{13}}{2}$

**Answer: (a) $\frac{3\sqrt{13}}{11}$ (b) $\frac{3\sqrt{13}}{2}$**

## 8 HCF and LCM

The HCF of two numbers is 2 and their LCM is 176.
One of the numbers is 22.
Work out the other number.

HCF × LCM = product of the two numbers
2 × 176 = 352
Other number = 352 ÷ 22 = 16
**Answer: 16**

## 9 Ratio and money transferred

Jamal and Evan share some money in the ratio 7 : 8.
Jamal gives Evan £4.
The ratio of their money is now 3 : 4.
How much money do they have altogether?

Let the amounts be 7k and 8k, so the total is 15k
Jamal now has 7k − 4, which is $\frac{3}{7}$ of the total
$7(7k - 4) = 3 \times 15k$
$49k - 28 = 45k$, so $4k = 28$ and $k = 7$
Total = 15 × 7 = £105
**Answer: £105**

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## 10 Bounds of speed, density and pressure

A force of 90 N acts on a surface, correct to 1 significant figure.
The area of the surface is 6 $m^2$, correct to the nearest whole number.
Work out the upper bound of the pressure.
Give your answer correct to 3 significant figures. (Calculator)

$85 \leq$ force $< 95$; $5.5 \leq$ area $< 6.5$
Upper bound of pressure = biggest force ÷ smallest area = 95 ÷ 5.5
= 17.2727... = 17.3 (3 s.f.)

**Answer: 17.3 $N/m^2$**