Day 290 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x - 30)°$, $(x + 35)°$, $(2x - 30)°$, $(x + 25)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$6x = 540$, so $6x = 540$
$x = 90$
(b) The angles are 90°, 60°, 125°, 150°, 115°
The largest is 150°
Answer: (a) x = 90 (b) 150°

2 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 2 : 1.
50% of the Year 10 students and 35% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 60 students in total: 40 Year 10 students and 20 Year 11 students
50% of 40 = 20 and 35% of 20 = 7
Together: 27 out of 60
$27 \div 60 \times 100 = 45\%$
Answer: 45%

3 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 13^2 - 12^2 = 25$
$AM = 5$ cm
$AB = 2 \times 5 = 10$ cm

(b) $\sin(\text{angle AOM}) = \frac{5}{13}$
Angle AOM = 22.62°, so angle AOB = $2 \times 22.62 = 45.2°$
Answer: (a) 10 cm (b) 45.2°

4 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 24 is taken. Work out how many come from Year 10.
(b) A different stratified sample has 33 from Year 8. Work out the size of this sample.

Total = 60 + 110 + 30 + 40 = 240

(a) Year 10: $\frac{40}{240} \times 24 = 4$

(b) Fraction sampled = $33 \div 110 = \frac{3}{10}$, so sample size = $\frac{3}{10} \times 240 = 72$

Answer: (a) 4 (b) 72

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5 Fraction problems

A fuel tank is $\frac{1}{10}$ full.

175 more litres of petrol are added and the fuel tank is then $\frac{4}{5}$ full.

Work out the capacity of the fuel tank.

Fraction added = $\frac{4}{5} - \frac{1}{10} = \frac{7}{10}$

$\frac{7}{10}$ of the capacity is 175 litres

Capacity = $175 \div \frac{7}{10} = 250$

Answer: 250 litres

6 Volume and surface area of composite solids

A solid is made from a cylinder of height 12 cm and a hemisphere. Both have radius 6 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cylinder: $\pi \times 6^2 \times 12 = 432\pi$

Hemisphere: $\frac{1}{2} \times \frac{4}{3} \times \pi \times 6^3 = 144\pi$

Total = $432\pi + 144\pi = 576\pi$ cm^3

Answer: 576π cm^3

7 Geometric sequences

The 2nd term of a geometric sequence is 48.
The 4th term of the sequence is 12.
All the terms of the sequence are positive.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 12 \div 48 = \frac{1}{4}$

$r = \sqrt{\frac{1}{4}} = \frac{1}{2}$ (positive, since all terms are positive)

(b) The 2nd term is $a \times r^1$: $a \times \frac{1}{2} = 48$

$a = 48 \div \frac{1}{2} = 96$

Answer: (a) $r = \frac{1}{2}$ (b) 96

8 Venn diagrams with algebra

The Venn diagram shows how many of 29 students have a tablet (T) or a laptop (L).
(a) Work out the value of x.
(b) A student is chosen at random from those who have a laptop. Work out the probability that this student also has a tablet.

(a) Total: $(x + 4) + x + (2x - 1) + 2 = 29$
$4x + 5 = 29$, so $x = 6$
(b) Students in the second circle: $x + 2x - 1 = 6 + 11 = 17$, of whom 6 are in both

Probability = $\frac{6}{17}$

Answer: (a) $x = 6$ (b) $\frac{6}{17}$

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9 Direct and inverse proportion

V is directly proportional to the cube of r.
When r = 10, V = 2000.
(a) Find a formula for V in terms of r.
(b) Find the value of V when r = 1.

(a) $V = kr^3$

Substitute r = 10 and V = 2000: $2000 = k \times 10^3$

$k = 2$

$V = 2r^3$

(b) $V = 21^3 = 2$

Answer: (a) $V = 2r^3$ (b) 2

10 Bounds and accuracy

a = 84 and b = 44, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$83.95 \leq a < 84.05$; $43.95 \leq b < 44.05$

Lower bound = 83.95 ÷ 44.05 = 1.90579...

Upper bound = 84.05 ÷ 43.95 = 1.91240...

Both bounds round to 1.91 to 3 significant figures, but not to 4 significant figures

Answer: 1.91 (3 s.f.)