Day 29 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 17 times the size of each exterior angle.
Work out the number of sides of the polygon.

Let each exterior angle be e. Then each interior angle is 17e.
Interior angle + exterior angle = 180°, so 17e + e = 180°
e = 180° ÷ 18 = 10°
Number of sides = 360° ÷ 10° = 36
Answer: 36

2 Equations with fractions

Solve $\frac{x-1}{5} - \frac{x-4}{3} = 1$

Multiply every term by 15: $3(x-1) - 5(x-4) = 15$
Expand: $-2x + 17 = 15$
$-2x = -2$
$x = 1$
Answer: x = 1

3 Weighted percentages

The ratio of boys to girls in a school is 3 : 2.
15% of the boys and 75% of the girls walk to school.
What percentage of all of these students walk to school?

Imagine 100 students in total: 60 boys and 40 girls
15% of 60 = 9 and 75% of 40 = 30
Together: 39 out of 100
39 ÷ 100 × 100 = 39%
Answer: 39%

4 Compound interest

Sofia invests £8,000.
The account pays compound interest of 5% a year for the first 2 years,
then 3.5% a year for the next 2 years.
Work out the total amount after 4 years, to the nearest penny. (Calculator)

Multipliers: 1.05 for 2 years, then 1.035 for 2 years
£8,000 × 1.05^2 × 1.035^2
= £9,448.20
Answer: £9,448.20

5 Area of a triangle using sine

Triangle ABC has AB = 12 cm and BC = 8 cm.
The area of triangle ABC is 24 cm^2.
Angle ABC is obtuse.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2}$ × AB × BC × sin B

$24 = \frac{1}{2} \times 12 \times 8 \times \sin B$

$\sin B = 24 \div 48 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is obtuse, so B = 180° − 30° = 150°

Answer: 150°

Day 29 **S Tier 10-a-day: Model Solutions**

6 Independent events and probability rules

A and B are two events.
P(A) = 0.4, P(B) = 0.4 and P(A or B) = 0.55
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.55 = 0.4 + 0.4 - P(A \cap B)$, so $P(A \cap B) = 0.25$
(b) $P(A) \times P(B) = 0.4 \times 0.4 = 0.16$
This is not equal to 0.25, so A and B are not independent
Answer: (a) 0.25 (b) No

7 Index equations

Solve $8^{2x-2} = 4^{x-1}$

Write both sides as powers of 2: $8 = 2^3$ and $4 = 2^2$
Equate the powers: $6x - 6 = 2x - 2$
$4x = 4$, so $x = 1$
Answer: $x = 1$

8 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $2\mathbf{a} + \mathbf{b}$, OQ = $5\mathbf{a} - 8\mathbf{b}$ and OR = $7\mathbf{a} - 14\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $3\mathbf{a} - 9\mathbf{b}$
QR = OR − OQ = $2\mathbf{a} - 6\mathbf{b}$
QR = $\frac{2}{3}$ PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 2
Answer: (b) 3 : 2

9 Perpendicular bisector

A is the point (2, 1) and B is the point (6, −1).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form y = mx + c.

Midpoint of AB = $(\frac{2+6}{2}, \frac{1+(-1)}{2}) = (4, 0)$

Gradient of AB = $\frac{-1-1}{6-2} = -\frac{1}{2}$

The perpendicular gradient is $-1 \div (-\frac{1}{2}) = 2$

Using the midpoint: $y = 2(x - 4)$
$y = 2x - 8$
Answer: $y = 2x - 8$

Day 29 **S Tier 10-a-day: Model Solutions**

10 Exact lengths and areas

A rectangle has area $(7 + 7\sqrt{2})$ cm^2 and length $(3 + \sqrt{2})$ cm.
Find the width of the rectangle.

Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\dfrac{7 + 7\sqrt{2}}{3 + \sqrt{2}}$

Multiply top and bottom by $(3 - \sqrt{2})$:

Numerator: $(7 + 7\sqrt{2})(3 - \sqrt{2}) = 7 + 14\sqrt{2}$

Denominator: $9 - 2 = 7$

Width = $1 + 2\sqrt{2}$

Answer: $1 + 2\sqrt{2}$ cm