**Day 289** **S Tier 10-a-day: Model Solutions**

## 1 Rationalising the denominator

Rationalise the denominator of $\frac{8}{4\sqrt{6}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{6}$:

$\frac{8}{4\sqrt{6}} = \frac{8\sqrt{6}}{24}$

$= \frac{\sqrt{6}}{3}$

**Answer:** $\frac{\sqrt{6}}{3}$

## 2 Geometric sequences

The 3rd term of a geometric sequence is 108.
The 6th term of the sequence is $-2916$.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^3 = -2916 \div 108 = -27$

$r = \sqrt[3]{-27} = -3$

(b) The 3rd term is $a \times r^2$: $a \times 9 = 108$

$a = 108 \div 9 = 12$

**Answer: (a) r = −3 (b) 12**

## 3 Probabilities with an unknown

A biased wheel stops on one of four outcomes with these probabilities.
A: 3x; B: x + 0.1; C: x + 0.1; D: 0.05
(a) Work out the value of x.
(b) The biased wheel is used 200 times. Work out an estimate for the number of times it stops on A.

(a) Probabilities add up to 1: $3x + x + 0.1 + x + 0.1 + 0.05 = 1$

$5x + 0.25 = 1$, so $5x = 0.75$

$x = 0.15$

(b) P(A) = $3 \times 0.15 = 0.45$, so $200 \times 0.45 = 90$

**Answer: (a) x = 0.15 (b) 90**

## 4 Trigonometric equations

Solve $3 \sin x° + 1 = 4$ for $0 \leq x \leq 360$

Rearrange: $3 \sin x° = 3$, so $\sin x° = 1$

From the graph of $y = \sin x°$, $\sin x° = 1$ when $x = 90$

**Answer: x = 90**

## 5 Fractions of fractions

Nia read $\frac{3}{4}$ of a book on Saturday.

On Sunday she read $\frac{3}{5}$ of the pages that were left.

She still had 18 pages to read.
How many pages are in the book?

After the first part, $\frac{1}{4}$ remains

After the second, $\frac{2}{5}$ of $\frac{1}{4}$ remains = $\frac{1}{10}$ of the total

$\frac{1}{10}$ of the total is 18

Total = $18 \div \frac{1}{10} = 180$

**Answer: 180 pages**

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## 6 Arc length and sector area

A sector of a circle has angle 90° and area $49\pi$ cm$^2$.
(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of $\pi$.

(a) Area = $\frac{90}{360} \times \pi \times r^2$, so $\frac{90}{360} \times r^2 = 49$

$r^2 = 49 \times \frac{360}{90} = 196$

$r = 14$ cm

(b) Arc length = $\frac{90}{360} \times 2 \times \pi \times 14 = 7\pi$ cm

**Answer: (a) 14 cm (b) $7\pi$ cm**

## 7 Compound interest

Hassan invests £5,500 at 6% per year compound interest.
After how many whole years will the investment first be worth more than £9,400?
(Calculator)

Multiplier = 1.06

After 9 years: $5500 \times 1.06^9 = 9292.13$

After 10 years: $5500 \times 1.06^{10} = 9849.66$

9849.66 is the first value above £9,400

**Answer: 10 years**

## 8 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.
(a) Work out the height VM of the pyramid.
(b) Work out the volume of the pyramid.
The diagram is not drawn accurately.

(a) $AC^2 = 16^2 + 16^2 = 512$, so $AM^2 = (AC \div 2)^2 = 128$

$VM^2 = VA^2 - AM^2 = 18^2 - 128 = 196$

$VM = 14$ cm

(b) Volume = $\frac{1}{3} \times 16^2 \times 14 = \frac{3584}{3}$ cm$^3$

**Answer: (a) 14 cm (b) $\frac{3584}{3}$ cm$^3$**

## 9 Estimating the median from grouped data

The table gives information about 110 masses.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 28, 58, 75, 81, 110

The median is the value at position $110 \div 2 = 55$

It is in the class 20 to 40, with 28 values before it, so we need 27 more of the 30 values in this class

Median $\approx 20 + \frac{27}{30} \times 20 = 38$

**Answer: 38**

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## 10 Tangent to a circle: proof

Show that the line $y = -x - 4$ is a tangent to the circle $x^2 + y^2 = 8$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = -x - 4$ into $x^2 + y^2 = 8$: $x^2 + (-x - 4)^2 = 8$

Expand: $2x^2 + 8x + 8 = 0$

$b^2 - 4ac = 8^2 - 4 \times 2 \times 8 = 64 - 64 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-8}{2 \times 2} = -2$, and $y = -(-2) - 4 = -2$. The point is $(-2, -2)$

**Answer: $(-2, -2)$**