Day 287 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $(p - q)^2 - (p + q)^2$
when $p = -1$, $q = 3$ and $r = 2$.

Substitute the values: $(-1 - 3)^2 - (-1 + 3)^2$
$= 16 - 4$
$= 12$
Answer: 12

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(3x + 30)°$
Angle PQR = $(3x + 10)°$
Angle PRS = $(5x + 55)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$5x + 55 = 6x + 40$
$x = 15$
(b) Angle PRS = 130° so angle PRQ = 180° − 130° = 50°
Answer: (a) x = 15 (b) 50°

3 Arithmetic sequences

The 2nd term of an arithmetic sequence is 16.
The 5th term of the sequence is 43.
(a) Find an expression for the nth term of the sequence.
(b) Is 160 a term of the sequence? You must show your working.

(a) Common difference = $(43 - 16) \div (5 - 2) = 9$
nth term = $9n + c$. For n = 2: $9 \times 2 + c = 16$, so $c = -2$
nth term = $9n - 2$
(b) Solve $9n - 2 = 160$: $9n = 162$, so $n = 18$
n is a positive whole number, so 160 is the 18th term
Answer: (a) 9n − 2 (b) Yes, it is the 18th term

4 Inverse proportion in context

9 machines can make a batch of bolts in 12 hours.
After 4 hours, 3 of the machines leave. The rest carry on working at the same rate.
How many more hours does it take to finish?

Total work = $9 \times 12 = 108$ machine-hours
Work done = $9 \times 4 = 36$, so work left = 72
Workers left = $9 - 3 = 6$
Time = $72 \div 6 = 12$ hours
Answer: 12 hours

5 Laws of indices

Given that $\frac{27^5 \times 3^3}{27^2} = 3^n$,
find the value of n.

$27 = 3^3$
Numerator: $3^{15} \times 3^3 = 3^{18}$
Denominator: 3^6
Subtract the powers: $18 - 6 = 12$
Answer: n = 12

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6 Trigonometric graphs

The diagram shows the graph of $y = a \cos bx$, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when $x = 340°$.

(a) The maximum value is 2 and the minimum is -2, so $a = 2$
The graph completes 3 full cycles between 0° and 360°, so $b = 3$
(b) $y = 2 \cos (3 \times 340°) = 2 \cos 1020°$
$\cos 300° = \frac{1}{2}$, so $y = 1$

Answer: (a) a = 2, b = 3 (b) y = 1

7 Finding a percentage rate

Dina invests £4,000 in an account paying compound interest.
The interest rate is the same percentage each year.
After 3 years the account holds £5,324.
Work out the annual percentage interest rate.

Multiplier for 3 years = £5,324 ÷ £4,000 = 1.331
Yearly multiplier = the cube root of 1.331 = 1.1
10% increase each year
Answer: 10%

8 Standard form problems

A factory makes 6.6×10^3 bottles each day.
It runs for 5×10^3 days.
How many bottles does it make?
Give your answer in standard form.

Total = $(6.6 \times 10^3) \times (5 \times 10^3)$
$6.6 \times 5 = 33$ and $10^3 \times 10^3 = 10^6$
$= 3.3 \times 10^7$
Answer: 3.3×10^7

9 Joint and inverse variation

F is directly proportional to m and inversely proportional to the square of d.
When m = 6 and d = 3, F = 2.
Work out the value of F when m = 1 and d = 1.

$F = \frac{km}{d^2}$
$2 = \frac{k \times 6}{9}$, so $k = 2 \times 9 \div 6 = 3$
$F = \frac{3 \times 1}{1} = 3$
Answer: F = 3

Day 287 **S Tier 10-a-day: Model Solutions**

10 Loci and regions

ABCD is a rectangle with AB = 6 cm and AD = 9 cm.
A point P inside the rectangle is such that AP ≤ 6 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 3 cm from A
AP ≤ 6 cm: P is inside the quarter circle of radius 6 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{3}{6}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 3 cm, height $6\sin 60° = 3\sqrt{3}$ cm, area $= \frac{1}{2} \times 3 \times 3\sqrt{3} = \frac{9}{2}\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 6^2 = 3\pi$

Area $= \frac{9}{2}\sqrt{3} + 3\pi$ cm^2

Answer: $\left(\frac{9}{2}\sqrt{3} + 3\pi\right)$ cm^2