Day 286 **S Tier 10-a-day: Model Solutions**

1 Exact trigonometric values

Work out the exact value of cos 45° + tan 45°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\cos 45° + \tan 45° = \frac{\sqrt{2}}{2} + 1$

$= 1 + \frac{\sqrt{2}}{2}$

Answer: $1 + \frac{\sqrt{2}}{2}$

2 Time zones

A flight leaves London at 15:50 (London time).
It takes 9 hours 10 minutes.
When it is 12:00 in London it is 14:00 in Cairo.
At what time, in Cairo time, does the flight land? Say if it is on a different day.

Arrival in London time = 15:50 + 9 h 10 min = 01:00 the next day
Cairo is 2 hours ahead of London, so add 2 hours
03:00 the next day
Answer: 03:00 the next day

3 Equations with fractions

Solve $\frac{x-4}{3} - \frac{x-3}{8} = -2$

Multiply every term by 24: $8(x-4) - 3(x-3) = -48$
Expand: $5x - 23 = -48$
$5x = -25$
$x = -5$
Answer: $x = -5$

4 Perpendicular bisector

A is the point (3, −3) and B is the point (9, −11).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (6, −7)

Gradient of AB = $-\frac{4}{3}$

Perpendicular gradient = $\frac{3}{4}$

$y - (-7) = \frac{3}{4}(x - 6)$

Rearrange to integer coefficients: $3x - 4y = 46$
Answer: $3x - 4y = 46$

5 Exponential growth and decay

There are 100 bacteria in a sample. The number of bacteria is multiplied by 3 every hour.
After how many hours will there first be more than 50,000 bacteria?

Number after n hours = 100×3^n
n = 5: 24,300
n = 6: 72,900
72,900 is the first value above 50,000
Answer: 6 hours

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6 Solving trigonometric equations

Solve $7 \sin x - 1 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\sin x = \frac{1}{7}$

Related acute angle: $\sin^{-1}(\frac{1}{7}) = 8.2°$

sine is positive in the 1st and 2nd quadrants, so $x = 8.2°$ and $x = 180° - 8.2°$
$x = 8.2°$ and $x = 171.8°$
Answer: $x = 8.2°$ or $x = 171.8°$

7 Percentages with an unknown rate

A population is increased by x% in one year and then increased by a further x% the next year.
Over the two years the population increases by 69%.
Work out the value of x.

Yearly multiplier $= 1 + \frac{x}{100}$

Two years: $(1 + \frac{x}{100})^2 = 1.69$

Take the square root: $1 + \frac{x}{100} = 1.3$

$\frac{x}{100} = 0.3$, so $x = 30$
Answer: $x = 30$

8 Composite functions: fg and gf

$f(x) = 3x - 3$ and $g(x) = x^2$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2) = 3x^2 - 3$
$gf(x) = g(3x - 3) = (3x - 3)^2 = 9x^2 - 18x + 9$
Set them equal and rearrange: $6x^2 - 18x + 12 = 0$
Divide by 6 and factorise: $(x - 2)(x - 1) = 0$
$x = 1$ or $x = 2$
Answer: $x = 1$ or $x = 2$

9 Tree diagrams and reverse probability

The probability that it rains on a day is 0.4.
If it rains, the probability that Ali is late for school is 0.2.
If it does not rain, the probability that Ali is late is 0.3.
(a) Work out the probability that Ali is late.
(b) Ali is late. Work out the probability that it rained.

(a) $P(\text{late}) = 0.4 \times 0.2 + 0.6 \times 0.3 = 0.08 + 0.18 = 0.26$
(b) $P(\text{rain and late}) = 0.08$

$P(\text{rain} \mid \text{late}) = 0.08 \div 0.26 = \frac{4}{13}$

Answer: (a) 0.26 (b) $\frac{4}{13}$

10 Surds with unknowns

Given that $(a + \sqrt{15})(b + \sqrt{15}) = 33 + 9\sqrt{15}$,
where a and b are positive integers with $a < b$, find the values of a and b.

$(a + \sqrt{15})(b + \sqrt{15}) = ab + (a + b)\sqrt{15} + 15$
Compare the $\sqrt{15}$ terms: $a + b = 9$
Compare the integer terms: $ab + 15 = 33$, so $ab = 18$
Two numbers with sum 9 and product 18 are 3 and 6
Answer: $a = 3, b = 6$