Day 285 **S Tier 10-a-day: Model Solutions**

1 Arc length and sector area

A sector of a circle has angle 216° and area $\frac{48}{5}\pi$ cm^2.

(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{216}{360} \times \pi \times r^2$, so $\frac{216}{360} \times r^2 = \frac{48}{5}$

$r^2 = \frac{48}{5} \times \frac{360}{216} = 16$

$r = 4$ cm

(b) Arc length = $\frac{216}{360} \times 2 \times \pi \times 4 = \frac{24}{5}\pi$ cm

Answer: (a) 4 cm (b) $\frac{24}{5}\pi$ cm

2 Recurring decimals

Without using a calculator, work out $0.\dot{9}\dot{1} \times 0.6\dot{6}\dot{3}$
Give your answer as a fraction in its simplest form.

$0.\dot{9}\dot{1} = \frac{91}{99}$

$0.6\dot{6}\dot{3} = \frac{73}{110}$

$\frac{91}{99} \times \frac{73}{110} = \frac{6643}{10890}$

Answer: $\frac{6643}{10890}$

3 Combined transformations

The point P(−4, 1) is reflected in the line $y = -x$.
The image is then reflected in the line $y = x$.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−1, 4)
After the second transformation: P'' = (4, −1)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−y, −x) → (−x, −y)
The single mapping is (x, y) → (−x, −y)
This is a rotation 180° about the origin
Answer: (a) (4, −1) (b) Rotation 180° about the origin

4 Iteration

Use the iteration formula $x_{n+1} = \sqrt{2x_n + 5}$ with $x_0 = 2$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt{2 \times 2 + 5} = 3.00000...$

$x_2 = \sqrt{2 \times x_1 + 5} = 3.31662...$

$x_3 = \sqrt{2 \times x_2 + 5} = 3.41075...$

Rounded: $x_1 = 3.000$, $x_2 = 3.317$, $x_3 = 3.411$

Answer: $x_1 = 3.000$, $x_2 = 3.317$, $x_3 = 3.411$

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5 Density of an alloy

An alloy is made by mixing 60 cm^3 of tin with 40 cm^3 of zinc.
The density of tin is 7.3 g/cm^3. The density of zinc is 7.1 g/cm^3.
Work out the density of the alloy.

Mass of tin = 60 × 7.3 = 438 g
Mass of zinc = 40 × 7.1 = 284 g
Total mass = 722 g, total volume = 100 cm^3
Density = 722 ÷ 100 = 7.22 g/cm^3
Answer: 7.22 g/cm^3

6 Fractional and negative indices

Work out $27^{-\frac{5}{3}}$

A negative power means take the reciprocal: $27^{-\frac{5}{3}} = (\frac{1}{27})^{\frac{5}{3}}$

The denominator 3 means the cube root: the cube root of $\frac{1}{27}$ is $\frac{1}{3}$

The numerator 5 means raise to the power 5: $(\frac{1}{3})^5 = \frac{1}{243}$

Answer: $\frac{1}{243}$

7 Composite functions: fg and gf

f(x) = 4x − 5 and g(x) = x^2 − 1
Solve fg(x) = gf(x)

fg(x) = f(x^2 − 1) = 4x^2 − 9
gf(x) = g(4x − 5) = (4x − 5)^2 − 1 = 16x^2 − 40x + 24
Set them equal and rearrange: 12x^2 − 40x + 33 = 0
Factorise: (2x − 3)(6x − 11) = 0
x = $\frac{3}{2}$ or x = $\frac{11}{6}$

Answer: x = $\frac{3}{2}$ or x = $\frac{11}{6}$

8 Tree diagrams and reverse probability

Machine A makes 40% of the bolts in a factory. Machine B makes the rest.
30% of the bolts from machine A are faulty.
40% of the bolts from machine B are faulty.
(a) Work out the probability that a bolt chosen at random is faulty.
(b) A faulty bolt is found. Work out the probability that it was made by machine A.

(a) P(faulty) = 0.4 × 0.3 + 0.6 × 0.4 = 0.12 + 0.24 = 0.36
(b) P(A and faulty) = 0.4 × 0.3 = 0.12

P(A | faulty) = 0.12 ÷ 0.36 = $\frac{1}{3}$

Answer: (a) 0.36 (b) $\frac{1}{3}$

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9 Tangent to a circle

A circle has equation $(x + 4)^2 + (y + 3)^2 = 100$.
The point P(2, 5) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is $(-4, -3)$. The gradient of the radius to P is $\frac{5-(-3)}{2-(-4)} = \frac{4}{3}$

The tangent is perpendicular to the radius, so its gradient is $-\frac{3}{4}$. Using P: $y - 5 = -\frac{3}{4}(x - 2)$

Multiply by 4: $4(y - 5) = -3(x - 2)$
Expand and rearrange: $3x + 4y = 26$
Answer: 3x + 4y = 26

10 Bounds and accuracy

a = 0.166 and b = 82.4, both correct to 3 significant figures.
Work out a × b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$0.1655 \leq a < 0.1665$; $82.35 \leq b < 82.45$
Lower bound = 0.1655 × 82.35 = 13.628925
Upper bound = 0.1665 × 82.45 = 13.727925
Both bounds round to 14 to 2 significant figures, but not to 3 significant figures
Answer: 14 (2 s.f.)