Day 284 **S Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x-4}{5} + \frac{x-6}{2} = -8$

Multiply every term by 10: $2(x-4) + 5(x-6) = -80$
Expand: $7x - 38 = -80$
$7x = -42$
$x = -6$
Answer: $x = -6$

2 Tests of divisibility

The 5-digit number 2726k is a multiple of 4.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 4 if its last two digits form a multiple of 4
Last two digits: 6k
Multiples of 4 of this form: 60, 64, 68
Answer: k = 0 or 4 or 8

3 Direct and inverse proportion

y is directly proportional to the square root of x.
When x = 4, y = 8.
(a) Find a formula for y in terms of x.
(b) Find the value of y when x = 1.

(a) $y = k\sqrt{x}$
Substitute x = 4 and y = 8: $8 = k \times \sqrt{4}$
$k = 4$
$y = 4\sqrt{x}$
(b) $y = 4\sqrt{1} = 4$
Answer: (a) $y = 4\sqrt{x}$ (b) 4

4 Fractions

Work out $(1\frac{1}{2} - \frac{2}{3}) \div 1\frac{1}{2}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions
$\frac{3}{2} - \frac{2}{3} = \frac{5}{6}$
$\frac{5}{6} \div \frac{3}{2} = \frac{5}{6} \times \frac{2}{3} = \frac{5}{9}$
Answer: $\frac{5}{9}$

5 Area and quadratic equations

The diagram shows a rectangle of length $(x - 2)$ cm, width $(x + 3)$ cm and area 50 cm^2.
(a) Show that $x^2 + x - 56 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(x - 2)(x + 3) = 50$
Expand: $x^2 + x - 6 = 50$, so $x^2 + x - 56 = 0$
(b) Factorise: $(x - 7)(x + 8) = 0$
$x = 7$ ($x = -8$ is not possible as a length would be negative)
Length = 5 cm and width = 10 cm
Perimeter = $2 \times (5 + 10) = 30$ cm
Answer: (b) 30 cm

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6 Recurring decimals

Show that $0.8\dot{7} = \frac{79}{90}$

Let x = 0.8777...
10x = 8.7777...
100x = 87.7777...
Subtract: 90x = 79
$x = \frac{79}{90}$

Answer: x = $\frac{79}{90}$

7 Area of a triangle using sine

Triangle ABC has AB = 20 cm and BC = 7 cm.
The area of triangle ABC is 35 cm^2.
Angle ABC is obtuse.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$35 = \frac{1}{2} \times 20 \times 7 \times \sin B$

$\sin B = 35 \div 70 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is obtuse, so B = 180° − 30° = 150°

Answer: 150°

8 Reverse percentages

The price of a tablet is increased by 25%.
Later the new price is reduced by 30% in a sale.
The sale price is £35.
Work out the price before the increase. (Calculator)

Multipliers: 1.25 and 0.7
Overall multiplier = 1.25 × 0.7 = 0.875
Original × 0.875 = £35
Original = £35 ÷ 0.875 = £40
Answer: £40

9 Equations with algebraic fractions

Solve $\frac{6}{x-2} - \frac{3}{x+4} = \frac{3}{4}$

Multiply every term by (x − 2)(x + 4): $6(x + 4) - 3(x - 2) = \frac{3}{4} \times (x - 2)(x + 4)$
Multiply by 4 to clear the fraction: $24(x + 4) - 12(x - 2) = 3(x - 2)(x + 4)$
Expand and rearrange so that one side is 0: $x^2 - 2x - 48 = 0$
Factorise: (x − 8)(x + 6) = 0
x = −6 or x = 8
Answer: x = −6 or x = 8

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10 Tree diagrams and reverse probability

The probability that it rains on a day is 0.6.
If it rains, the probability that Ali is late for school is 0.8.
If it does not rain, the probability that Ali is late is 0.4.
(a) Work out the probability that Ali is late.
(b) Ali is late. Work out the probability that it rained.

(a) P(late) = 0.6 × 0.8 + 0.4 × 0.4 = 0.48 + 0.16 = 0.64
(b) P(rain and late) = 0.48

P(rain | late) = 0.48 ÷ 0.64 = $\frac{3}{4}$

Answer: (a) 0.64 (b) $\frac{3}{4}$