Day 283 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 38 is taken. Work out how many come from Year 9.
(b) A different stratified sample has 30 from Year 7. Work out the size of this sample.

Total = 200 + 240 + 140 + 180 = 760

(a) Year 9: $\frac{140}{760} \times 38 = 7$

(b) Fraction sampled = $30 \div 200 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 760 = 114$

Answer: (a) 7 (b) 114

2 Algebraic fractions: adding and subtracting

Write $\frac{x-4}{3x+4} + \frac{5}{x+2}$ as a single fraction.
Simplify the numerator.

The common denominator is (3x + 4)(x + 2)

$$\frac{x-4}{3x+4} + \frac{5}{x+2} = \frac{(x-4)(x+2) + 5(3x+4)}{(3x+4)(x+2)}$$

Expand the numerator: $x^2 + 13x + 12$

$$\frac{x^2 + 13x + 12}{(3x+4)(x+2)}$$

Answer: $\mathbf{\frac{x^2 + 13x + 12}{(3x+4)(x+2)}}$

3 Rationalising the denominator

Rationalise the denominator of $\frac{14}{4\sqrt{7}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{7}$:

$$\frac{14}{4\sqrt{7}} = \frac{14\sqrt{7}}{28}$$

$$= \frac{\sqrt{7}}{2}$$

Answer: $\mathbf{\frac{\sqrt{7}}{2}}$

4 Probabilities with an unknown

A biased wheel stops on one of four outcomes with these probabilities.
A: x; B: x + 0.1; C: 2x; D: 0.1
(a) Work out the value of x.
(b) The biased wheel is used 80 times. Work out an estimate for the number of times it stops on A.

(a) Probabilities add up to 1: x + x + 0.1 + 2x + 0.1 = 1
4x + 0.2 = 1, so 4x = 0.8
x = 0.2
(b) P(A) = x = 0.2, so 80 × 0.2 = 16
Answer: (a) x = 0.2 (b) 16

5 HCF and LCM

(a) Write 42 and 3024 as products of prime factors.
(b) Find the HCF and the LCM of 42 and 3024.
Give your answers as numbers.

(a) $42 = 2 \times 3 \times 7$ and $3024 = 2^4 \times 3^3 \times 7$
(b) HCF = $2 \times 3 \times 7 = 42$
LCM = $2^4 \times 3^3 \times 7 = 3024$
Answer: (a) $2 \times 3 \times 7$; $2^4 \times 3^3 \times 7$ (b) HCF = 42, LCM = 3024

Day 283 **S Tier 10-a-day: Model Solutions**

6 Direct and inverse proportion

V is directly proportional to the square root of r.
When r = 36, V = 42.
(a) Find a formula for V in terms of r.
(b) Find the value of V when r = 1.

(a) $V = k\sqrt{r}$
Substitute r = 36 and V = 42: $42 = k \times \sqrt{36}$
k = 7
$V = 7\sqrt{r}$
(b) $V = 7\sqrt{1} = 7$
Answer: (a) $V = 7\sqrt{r}$ (b) 7

7 Income tax

In one country, income tax is worked out like this:
The first £15,000 of income is not taxed.
The next £20,000 is taxed at 20%.
All income above that is taxed at 45%.
Kofi earns £40,000 in a year.
Work out how much income tax Kofi pays.

Taxed at 20%: £20,000
20% of £20,000 = £4,000
Taxed at 45%: £40,000 − £15,000 − £20,000 = £5,000
45% of £5,000 = £2,250
Total tax = £4,000 + £2,250 = £6,250
Answer: £6,250

8 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 3**b**, OQ = **a** + **b** and OR = 3**a** − 3**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = **a** − 2**b**
QR = OR − OQ = 2**a** − 4**b**
QR = 2 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 1 : 2
Answer: (b) 1 : 2

9 Estimating the mean from a histogram table

The table gives information about the times of some people.
Work out an estimate for the mean time.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 13, 11, 38, 12
Mid-points: 5, 15, 30, 50
Σfm = 13×5 + 11×15 + 38×30 + 12×50 = 1970, Σf = 74
Mean = 1970 ÷ 74 = 26.6
Answer: 26.6

Day 283 **S Tier 10-a-day: Model Solutions**

10 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out the length of AB, marked x, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $x^2 = 12^2 + 10^2 - 2 \times 12 \times 10 \times \cos 70°$
$x^2 = 161.92$
$x = 12.7$ cm (1 d.p.)

(b) Area $= \frac{1}{2} \times 12 \times 10 \times \sin 70° = 56.4$ cm^2 (3 s.f.)

Answer: (a) 12.7 cm (b) 56.4 cm^2