Day 282 **S Tier 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 6 numbers is 146.
One number is removed and the mean of the remaining 5 numbers is 172.4.
Work out the number that was removed.

Total of the 6 numbers = 6 × 146 = 876
Total of the 5 remaining = 5 × 172.4 = 862
Number removed = 876 − 862 = 14
Answer: 14

2 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 12 cm, DB = 4 cm and the area of triangle ADE is 216 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 12 + 4 = 16 cm

Length scale factor = 16 ÷ 12 = $\frac{4}{3}$

Area scale factor = $(\frac{4}{3})^2 = \frac{16}{9}$

Area of triangle ABC = 216 × $\frac{16}{9}$ = 384 cm^2

Trapezium = 384 − 216 = 168 cm^2
Answer: 168 cm^2

3 Estimating the mean from a histogram table

The table gives information about the ages of some people.
Work out an estimate for the mean age.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 8, 4, 10, 18, 36
Mid-points: 12.5, 17.5, 25, 35, 50
Σfm = 8×12.5 + 4×17.5 + 10×25 + 18×35 + 36×50 = 2850, Σf = 76
Mean = 2850 ÷ 76 = 37.5
Answer: 37.5

4 Error intervals

The width of a table is 7300 cm, correct to the nearest 100.
Write down the error interval for the width of a table, using w.

The value has been rounded to the nearest 100
Half of this is 50, so go 50 below and 50 above
7300 − 50 = 7250 and 7300 + 50 = 7350
7250 ≤ w < 7350
Answer: 7250 ≤ w < 7350

5 Density of an alloy

An alloy is made by mixing 120 cm^3 of aluminium with 120 cm^3 of nickel.
The density of aluminium is 2.7 g/cm^3. The density of nickel is 8.9 g/cm^3.
Work out the density of the alloy.

Mass of aluminium = 120 × 2.7 = 324 g
Mass of nickel = 120 × 8.9 = 1068 g
Total mass = 1392 g, total volume = 240 cm^3
Density = 1392 ÷ 240 = 5.8 g/cm^3
Answer: 5.8 g/cm^3

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6 Inverse functions

$f(x) = \frac{2x + 6}{x - 2}$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(12)$.

(a) Let $y = \frac{2x + 6}{x - 2}$. Then $y(x - 2) = 2x + 6$

$xy - 2y = 2x + 6$

Collect x terms: $xy - 2x = 6 + 2y$

Factorise: $x(y - 2) = 2y + 6$, so $x = \frac{2y + 6}{y - 2}$

$f^{-1}(x) = \frac{2x + 6}{x - 2}$

(b) $f^{-1}(12) = \frac{2 \times 12 + 6}{1 \times 12 - 2} = \frac{30}{10} = 3$

Answer: (a) $f^{-1}(x) = \frac{2x + 6}{x - 2}$ (b) 3

7 Enlargement

Triangle P has vertices A(−5, 3), B(1, 1) and C(−7, 7).
Triangle Q has vertices A'(−3, 3), B'(0, 2) and C'(−4, 5).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = 6, x-step from A' to B' = 3

Scale factor = $3 \div 6 = \frac{1}{2}$

The centre C satisfies: $C + \frac{1}{2}(A - C) = A'$, so $(1 - \frac{1}{2})C = A' - \frac{1}{2}A$

x: $\frac{1}{2} \times x = (-3) - \frac{1}{2} \times (-5) = -\frac{1}{2}$, so $x = -1$

y: $\frac{1}{2} \times y = 3 - \frac{1}{2} \times 3 = \frac{3}{2}$, so $y = 3$

Answer: Enlargement, scale factor $\frac{1}{2}$, centre $(-1, 3)$

8 Tangent to a circle: proof

Show that the line $y = x - 6$ is a tangent to the circle $x^2 + y^2 = 18$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = x - 6$ into $x^2 + y^2 = 18$: $x^2 + (x - 6)^2 = 18$

Expand: $2x^2 - 12x + 18 = 0$

$b^2 - 4ac = (-12)^2 - 4 \times 2 \times 18 = 144 - 144 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-(-12)}{2 \times 2} = 3$, and $y = 3 - 6 = -3$. The point is $(3, -3)$

Answer: $(3, -3)$

9 HCF and LCM

The HCF of two numbers is 3 and their LCM is 198.
The smaller number is 18.
Work out the larger number.

HCF × LCM = product of the two numbers

$3 \times 198 = 594$

Larger number = $594 \div 18 = 33$

Answer: 33

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10 Cone made from a sector

A sector of a circle has radius 8 cm and angle 180°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{180}{360} \times 2\pi \times 8 = 8\pi$ cm

This is the circumference of the base: $2\pi r = 8\pi$, so r = 4 cm

(b) Slant height = 8 cm, so height$^2 = 8^2 - 4^2 = 48$, height = $4\sqrt{3}$ cm

Volume = $\frac{1}{3} \times \pi \times 4^2 \times 4\sqrt{3} = \frac{64\sqrt{3}}{3}\pi$ cm^3

Answer: (a) 4 cm (b) $\frac{64\sqrt{3}}{3}\pi$ cm^3