Day 281 **S Tier 10-a-day: Model Solutions**

1 Fraction problems

A bag contains balloons.

$\frac{1}{4}$ of the balloons are pink and $\frac{2}{5}$ of the balloons are silver.

The rest of the balloons, 21 of them, are gold.

How many silver balloons are there?

Fraction that are gold = $1 - \frac{1}{4} - \frac{2}{5} = \frac{7}{20}$

$\frac{7}{20}$ of the total is 21

Total = $21 \div \frac{7}{20} = 60$

Silver balloons = $\frac{2}{5}$ of 60 = 24

Answer: 24

2 Similar triangles with algebra

In the diagram, DE is parallel to BC.

AD = x cm, DB = (x + 5) cm, DE = 2 cm and BC = 6 cm.

(a) Find the value of x.

(b) Work out the length of AB.

The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 5} = \frac{2}{6}$

$6x = 2(2x + 5)$

$6x = 4x + 10$

$2x = 10$

$x = 5$

(b) AB = 2x + 5 = 15 cm

Answer: (a) x = 5 (b) 15 cm

3 Percentage change

The population of a village increased by 10% in 2023.

In 2024 the population increased by a further 5%.

Work out the overall percentage increase over the two years.

Multipliers: 1.1 and 1.05

Overall multiplier = $1.1 \times 1.05 = 1.155$

$1.155 - 1 = 0.155$, so an overall increase of 15.5%

Answer: 15.5% increase

4 Inverse proportion in context

24 painters can paint a hall in 10 days.

After 3 days, 3 of the painters leave. The rest carry on working at the same rate.

How many more days does it take to finish?

Total work = $24 \times 10 = 240$ painter-days

Work done = $24 \times 3 = 72$, so work left = 168

Workers left = $24 - 3 = 21$

Time = $168 \div 21 = 8$ days

Answer: 8 days

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5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 8 cm, DB = 12 cm and the area of triangle ADE is 80 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 8 + 12 = 20 cm

Length scale factor = $20 \div 8 = \frac{5}{2}$

Area scale factor = $(\frac{5}{2})^2 = \frac{25}{4}$

Area of triangle ABC = $80 \times \frac{25}{4} = 500$ cm^2

Trapezium = $500 - 80 = 420$ cm^2

Answer: 420 cm^2

6 Cumulative frequency graphs

The cumulative frequency graph gives information about the times of 100 students to complete a puzzle.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 100 had a time greater than 55.

(a) Median at cumulative frequency 50, so median = 40
Lower quartile at 25, so LQ = 30. Upper quartile at 75, so UQ = 50
Interquartile range = 50 − 30 = 20
(b) At 55, cumulative frequency = 85
Number greater = 100 − 85 = 15
Answer: (a) median 40, IQR 20 (b) 15

7 Equal roots and real roots

The equation $kx^2 + 8x + 2 = 0$ has equal roots.
Find the value of k.

Equal roots means $b^2 - 4ac = 0$
$(8)^2 - 4 \times k \times 2 = 0$, so $64 = 8k$
$k = 8$
Answer: k = 8

8 Probability from a frequency tree

A factory makes 10,000 parts. 1% of them are faulty.
A scanner flags 80% of faulty parts and wrongly flags 2% of parts that are not faulty.
A part is chosen at random from the parts flagged by the scanner.
Work out the probability that this part is faulty.

1% of 10,000 = 100 are faulty and 9,900 are not
Flagged and genuine: 100 × 80% = 80
Flagged but not: 9,900 × 2% = 198
Total flagged or positive = 80 + 198 = 278

Probability = $80 \div 278 = \frac{40}{139}$

Answer: $\frac{40}{139}$

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9 Bounds of roots

The area of a circle is 220 cm^2, correct to the nearest cm^2.
Work out the lower bound of the radius of the circle.
Give your answer correct to 3 significant figures. (Calculator)

$219.5 \leq \text{area} < 220.5$

Area = πr^2, so r = $\sqrt{\frac{\text{area}}{\pi}}$

Lower bound = $\sqrt{\frac{219.5}{\pi}}$ = 8.35877... = 8.36 (3 s.f.)

Answer: 8.36 cm

10 Circle and line intersections

Solve the simultaneous equations

$x^2 + y^2 = 52$

$y = x - 2$

Substitute into the first equation: $x^2 + (x - 2)^2 = 52$

Expand: $2x^2 - 4x - 48 = 0$

Divide by 2: $x^2 - 2x - 24 = 0$

Factorise: $(x + 4)(x - 6) = 0$, so $x = -4$ or $x = 6$

$y = x - 2$ gives $y = -6$ and $y = 4$

Answer: $x = -4$, $y = -6$ and $x = 6$, $y = 4$