Day 280 **S Tier 10-a-day: Model Solutions**

1 Quadratic inequalities

Solve $x^2 + 9x + 8 \geq 0$

Solve $x^2 + 9x + 8 = 0$: $(x + 8)(x + 1) = 0$, so $x = -8$ or $x = -1$
The graph is a U shape crossing the x-axis at -8 and -1; it is above the axis outside the roots
$x \leq -8$ or $x \geq -1$
Answer: $x \leq -8$ or $x \geq -1$

2 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = $(4x - 20)°$
Angle ABC = $(2x - 20)°$
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
$(4x - 20) + 2(2x - 20) = 180$
$8x - 60 = 180$, so $8x = 240$
$x = 30$
(b) Angle BAC = $4 \times 30 - 20 = 100°$
Answer: (a) $x = 30$ (b) $100°$

3 Mean of a changing set

The mean of 9 numbers is 71.
One number is removed and the mean of the remaining 8 numbers is 75.
Work out the number that was removed.

Total of the 9 numbers = $9 \times 71 = 639$
Total of the 8 remaining = $8 \times 75 = 600$
Number removed = $639 - 600 = 39$
Answer: 39

4 Expanding three brackets

Expand and simplify $(3x - 6)(x - 4)(x + 6)$

Expand the first two brackets: $(x - 4)(x + 6) = x^2 + 2x - 24$
Multiply by $(3x - 6)$: $3x(x^2 + 2x - 24) - 6(x^2 + 2x - 24)$
$= 3x^3 - 84x + 144$
Answer: $3x^3 - 84x + 144$

5 Prime factors and indices

Write $30^2 \times 15^4$ as a product of powers of prime numbers.

$30 = 2 \times 3 \times 5$, $15 = 3 \times 5$
$30^2 = 2^2 \times 3^2 \times 5^2$
$15^4 = 3^4 \times 5^4$
Add the powers of each prime: $2^2 \times 3^6 \times 5^6$
Answer: $2^2 \times 3^6 \times 5^6$

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6 Area of a segment

The circle has centre O and radius 13 cm. A and B are points on the circle and angle AOB = 140°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{140}{360} \times \pi \times 13^2 = 206.47$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 13 \times 13 \times \sin 140° = 54.316$ cm^2

Segment = sector − triangle = 206.47 − 54.316 = 152.16

= 152 cm^2 (3 s.f.)

Answer: 152 cm^2

7 Winning a match

Carlos and Dina play a match. The first to win 2 tennis sets wins the match.

The probability that Carlos wins any one of the tennis sets is $\frac{7}{10}$.

Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Carlos wins then loses) = $\frac{7}{10} \times \frac{3}{10} = \frac{21}{100}$

P(Carlos loses then wins) = $\frac{3}{10} \times \frac{7}{10} = \frac{21}{100}$

Total = $2 \times \frac{21}{100} = \frac{21}{50}$

Answer: $\frac{21}{50}$

8 Line and curve intersections

Solve the simultaneous equations

$y = x^2 - 4$

$y = -4x - 4$

Substitute: $x^2 - 4 = -4x - 4$

Rearrange: $x^2 + 4x = 0$

Factorise: $(x + 4)x = 0$, so $x = -4$ or $x = 0$

When $x = -4$: $y = -4 \times (-4) - 4 = 12$

When $x = 0$: $y = -4 \times 0 - 4 = -4$

Answer: $x = -4$, $y = 12$ and $x = 0$, $y = -4$

9 Cone made from a sector

A sector of a circle has radius 24 cm and angle 90°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{90}{360} \times 2\pi \times 24 = 12\pi$ cm

This is the circumference of the base: $2\pi r = 12\pi$, so $r = 6$ cm

(b) Slant height = 24 cm, so height$^2 = 24^2 - 6^2 = 540$, height = $6\sqrt{15}$ cm

Volume = $\frac{1}{3} \times \pi \times 6^2 \times 6\sqrt{15} = 72\sqrt{15}\pi$ cm^3

Answer: (a) 6 cm (b) $72\sqrt{15}\pi$ cm^3

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10 Standard form problems

Light travels at 3×10^8 metres per second.
A planet is 1.83×10^{15} metres from a spacecraft.
How many seconds does a light signal take to travel from the planet to the spacecraft?
Give your answer in standard form.

Time = distance ÷ speed

$(1.83 \times 10^{15}) \div (3 \times 10^8)$

$1.83 \div 3 = 0.61$ and $10^{15} \div 10^8 = 10^7$

$= 6.1 \times 10^6$

Answer: 6.1×10^6 seconds