Day 28 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 11 times the size of each exterior angle.
Work out the number of sides of the polygon.

Let each exterior angle be e. Then each interior angle is 11e.
Interior angle + exterior angle = 180°, so 11e + e = 180°
e = 180° ÷ 12 = 15°
Number of sides = 360° ÷ 15° = 24
Answer: 24

2 Trigonometric graphs

The diagram shows the graph of $y = a \cos x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At x = 0, cos x° = 1, so a + c = −8
At x = 180, cos x° = −1, so −a + c = −6
Add: 2c = −14, so c = −7
Subtract: 2a = −2, so a = −1
Answer: a = −1, c = −7

3 Capture and recapture

100 deer in a forest are caught, marked and put back.
Later, a second sample of 80 deer is caught. 5 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 5 out of 80

This should match the proportion marked in the whole population: $\frac{5}{80} = \frac{100}{N}$

N = 100 × 80 ÷ 5 = 1600
(b) No deer are born, die or leave between the samples, and the deer mix evenly
Answer: (a) 1600 (b) The population stays the same between samples

4 Arithmetic sequences

The 6th term of an arithmetic sequence is 42.
The 10th term of the sequence is 58.
(a) Find an expression for the nth term of the sequence.
(b) Is 163 a term of the sequence? You must show your working.

(a) Common difference = (58 − 42) ÷ (10 − 6) = 4
nth term = 4n + c. For n = 6: 4 × 6 + c = 42, so c = 18
nth term = 4n + 18

(b) Solve 4n + 18 = 163: 4n = 145, so $n = \frac{145}{4}$

n is not a whole number, so 163 is not a term
Answer: (a) 4n + 18 (b) No

5 Solving trigonometric equations

Solve $\tan x = \sqrt{3}$ for $0° \leq x \leq 360°$.
Do not use a calculator.

The related acute angle satisfies $\tan \theta = \sqrt{3}$, so θ = 60°
tangent is positive in the 1st and 3rd quadrants, so x = 60° and x = 180° + 60°
x = 60° and x = 240°
Answer: x = 60° or x = 240°

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6 Powers of standard form

Work out $(2.8 \times 10^1)^3$
Give your answer in standard form.

$(2.8 \times 10^1)^3 = 2.8^3 \times (10^1)^3$
$2.8^3 = 21.952$ and $(10^1)^3 = 10^3$
$21.952 \times 10^3 = 2.1952 \times 10^4$
Answer: 2.1952×10^4

7 Percentage change and proportion

The kinetic energy of a moving car is directly proportional to the square of its speed.
Its speed is increased by 50%.
Work out the percentage change in the kinetic energy of a moving car.

The speed is multiplied by 1.5
The multiplier for the kinetic energy of a moving car is $1.5^2 = 2.25$
2.25 is an increase of 125%
Answer: 125% increase

8 Loci and regions

ABCD is a rectangle with AB = 7 cm and AD = 5 cm.
A point P inside the rectangle is such that AP ≤ 2 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 2 cm: P is inside the quarter circle of radius 2 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 2^2 = \frac{1}{2}\pi$ cm^2
Answer: $\frac{1}{2}\pi$ cm^2

9 Speed and a moving train

A train travels at a constant speed of 90 km/h.
It takes 60 seconds to go completely through a tunnel which is 1,200 m long.
Work out the length of the train, in metres.

Speed = 90 km/h = 90,000 ÷ 3600 = 25 m/s
Distance travelled in 60 s = 25 × 60 = 1,500 m
This is the length of the tunnel plus the length of the train
Train = 1,500 − 1,200 = 300 m
Answer: 300 m

10 Finding a percentage rate

A machine was bought for £9,100.
Each year its value decreases by the same percentage.
After 2 years it is worth £5,824.
Work out the percentage decrease each year.

Multiplier for 2 years = £5,824 ÷ £9,100 = 0.64
Yearly multiplier = the square root of 0.64 = 0.8
20% decrease each year
Answer: 20%