Day 279 **S Tier 10-a-day: Model Solutions**

1 Distance-time graphs

The graph shows Amir's cycle ride to a café and back.
(a) Work out Amir's speed on the way back.
(b) Work out Amir's average speed for the whole ride, including the stop.

(a) Return: 20 km in 2.5 hours, so speed = 20 ÷ 2.5 = 8 km/h
(b) Total distance = 2 × 20 = 40 km, total time = 5 hours
Average speed = 40 ÷ 5 = 8 km/h
Answer: (a) 8 km/h (b) 8 km/h

2 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 65°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 65°
Angle AOB = 180 − 2 × 65 = 50°
(b) The angle at the circumference is half the angle at the centre: 50 ÷ 2 = 25°
Answer: (a) 50° (b) 25°

3 Pressure

A metal crate in the shape of a cuboid weighs 480 N.
Its smallest face has area 200 cm^2. Its largest face has area 1,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 200 cm^2 = 0.02 m^2 and 1,000 cm^2 = 0.1 m^2
(a) Greatest pressure is on the smallest face: 480 ÷ 0.02 = 24,000 N/m^2
(b) Least pressure is on the largest face: 480 ÷ 0.1 = 4,800 N/m^2
Answer: (a) 24,000 N/m^2 (b) 4,800 N/m^2

4 Fraction problems

A bag contains sweets.
$\frac{3}{10}$ of the sweets are orange and $\frac{1}{3}$ of the sweets are lemon.
The rest of the sweets, 22 of them, are strawberry.
How many orange sweets are there?

Fraction that are strawberry = $1 - \frac{3}{10} - \frac{1}{3} = \frac{11}{30}$

$\frac{11}{30}$ of the total is 22

Total = $22 \div \frac{11}{30} = 60$

Orange sweets = $\frac{3}{10}$ of 60 = 18

Answer: 18

5 Compound interest

Callum invests £6,500 at 8% per year compound interest.
After how many whole years will the investment first be worth more than £11,300?
(Calculator)

Multiplier = 1.08
After 7 years: $6500 \times 1.08^7 = 11139.86$
After 8 years: $6500 \times 1.08^8 = 12031.05$
12031.05 is the first value above £11,300
Answer: 8 years

Day 279 **S Tier 10-a-day: Model Solutions**

6 Radius from chord and height of arc

A circular arc has a chord AB of length 42 cm.
The greatest distance between the chord and the arc is 7 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 7

AM = 42 ÷ 2 = 21. Pythagoras in triangle OMA: $r^2 = (r - 7)^2 + 21^2$

$r^2 = r^2 - 14r + 49 + 441$

14r = 490, so r = 35 cm

Answer: 35 cm

7 Geometric sequences

The 3rd term of a geometric sequence is 16.
The 5th term of the sequence is 64.
All the terms of the sequence are positive.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 64 \div 16 = 4$

$r = \sqrt{4} = 2$ (positive, since all terms are positive)

(b) The 3rd term is $a \times r^2$: $a \times 4 = 16$

a = 16 ÷ 4 = 4

Answer: (a) r = 2 (b) 4

8 Angles of elevation

A point A is on level ground in line with the base of a mast. (Calculator)
The angle of elevation of the top of the mast from A is 43°.
B is a point on the same line, 20 m further from the mast than A.
The angle of elevation of the top of the mast from B is 16°.
Work out the height of the mast. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 43° = \frac{h}{x}$, so $x = h \div \tan 43°$

From B: $\tan 16° = \frac{h}{x + 20}$, so $x + 20 = h \div \tan 16°$

Subtract: $20 = h \div \tan 16° - h \div \tan 43° = h(3.4874 - 1.0724)$

h = 20 ÷ 2.4150 = 8.3 m (1 d.p.)

Answer: 8.3 m

9 Counting factors

$N = 2^5 \times 3^2 \times 5$
How many of the factors of N are odd numbers?

An odd factor contains no 2s
Ignore the 2 and count the factors of the rest: 3 × 2 = 6

Answer: 6

Day 279 **S Tier 10-a-day: Model Solutions**

10 Perpendicular bisector

A is the point (0, −1) and B is the point (6, 3).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form y = mx + c.

Midpoint of AB = $(\frac{0+6}{2}, \frac{-1+3}{2}) = (3, 1)$

Gradient of AB = $\frac{3-(-1)}{6-0} = \frac{2}{3}$

The perpendicular gradient is $-1 \div \frac{2}{3} = -\frac{3}{2}$

Using the midpoint: $y - 1 = -\frac{3}{2}(x - 3)$

$y = -\frac{3}{2}x + \frac{11}{2}$

Answer: $y = -\frac{3}{2}x + \frac{11}{2}$