**Day 278** **S Tier 10-a-day: Model Solutions**

## 1 Standard form problems

A virus has a diameter of $2 \times 10^{-7}$ metres.
Viruses are placed in a straight line, touching, to make a length of $1.86 \times 10^{-3}$ metres.
How many viruses are in the line?
Give your answer in standard form.

Number = total length ÷ diameter
$(1.86 \times 10^{-3}) \div (2 \times 10^{-7})$
$1.86 \div 2 = 0.93$ and $10^{-3} \div 10^{-7} = 10^{4}$
$= 9.3 \times 10^{3}$
**Answer: $9.3 \times 10^{3}$**

## 2 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 35 is taken. Work out how many come from Year 10.
(b) A different stratified sample has 30 from Year 8. Work out the size of this sample.

Total = 175 + 250 + 250 + 200 = 875
(a) Year 10: $\frac{200}{875} \times 35 = 8$
(b) Fraction sampled = $30 \div 250 = \frac{3}{25}$, so sample size = $\frac{3}{25} \times 875 = 105$
**Answer: (a) 8 (b) 105**

## 3 Factorising fully

Factorise fully $x^4 - 256$

$x^4 - 256 = (x^2)^2 - 16^2 = (x^2 - 16)(x^2 + 16)$
$x^2 - 16$ is also a difference of two squares: $(x - 4)(x + 4)$
**Answer: $(x^2 + 16)(x - 4)(x + 4)$**

## 4 Arc length and sector area

A sector of a circle has angle 150° and area $\frac{245}{12}\pi$ cm$^2$.
(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of $\pi$.

(a) Area = $\frac{150}{360} \times \pi \times r^2$, so $\frac{150}{360} \times r^2 = \frac{245}{12}$
$r^2 = \frac{245}{12} \times \frac{360}{150} = 49$
r = 7 cm
(b) Arc length = $\frac{150}{360} \times 2 \times \pi \times 7 = \frac{35}{6}\pi$ cm
**Answer: (a) 7 cm (b) $\frac{35}{6}\pi$ cm**

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## 5 Iteration: rearranging equations

(a) Show that the equation $x^2 - 9x - 16 = 0$ can be rearranged to give $x = 9 + \frac{16}{x}$ (Calculator)

(b) Use the iteration formula $x_{n+1} = 9 + \frac{16}{x_n}$ with $x_0 = 10$ to work out $x_1$ and $x_2$. Give each answer correct to 3 decimal places.

(a) Rearrange $x^2 - 9x - 16 = 0$: $x^2 - 9x = 16$
Factorise: $x(x - 9) = 16$
Divide by x: $x - 9 = \frac{16}{x}$, so $x = 9 + \frac{16}{x}$

(b) $x_1 = 9 + \frac{16}{10} = 10.60000...$

$x_2 = 9 + \frac{16}{x_1} = 10.50943...$

$x_1 = 10.600$ and $x_2 = 10.509$ (3 d.p.)
**Answer: (b) $x_1 = 10.600$, $x_2 = 10.509$**

## 6 Cumulative frequency graphs

The cumulative frequency graph gives information about the times of 100 students to complete a puzzle.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 100 had a time greater than 15.

(a) Median at cumulative frequency 50, so median = 40
Lower quartile at 25, so LQ = 30. Upper quartile at 75, so UQ = 50
Interquartile range = 50 − 30 = 20
(b) At 15, cumulative frequency = 7
Number greater = 100 − 7 = 93
**Answer: (a) median 40, IQR 20 (b) 93**

## 7 Income tax

In one country, income tax is worked out like this:
The first £12,500 of income is not taxed.
The next £20,000 is taxed at 20%.
All income above that is taxed at 45%.
Priya earns £42,500 in a year.
Work out how much income tax Priya pays.

Taxed at 20%: £20,000
20% of £20,000 = £4,000
Taxed at 45%: £42,500 − £12,500 − £20,000 = £10,000
45% of £10,000 = £4,500
Total tax = £4,000 + £4,500 = £8,500
**Answer: £8,500**

## 8 Estimating the mean from a histogram table

The table gives information about the times of some people.
Work out an estimate for the mean time.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 4, 6, 32, 40
Mid-points: 5, 15, 30, 50
$\Sigma fm = 4\times5 + 6\times15 + 32\times30 + 40\times50 = 3070$, $\Sigma f = 82$
Mean = 3070 ÷ 82 = 37.4
**Answer: 37.4**

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## 9 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $2\mathbf{a} - 2\mathbf{b}$, OQ = $-\mathbf{a} - \mathbf{b}$ and OR = $-13\mathbf{a} + 3\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-3\mathbf{a} + \mathbf{b}$
QR = OR − OQ = $-12\mathbf{a} + 4\mathbf{b}$
QR = 4 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 1 : 4
**Answer: (b) 1 : 4**

## 10 Algebraic proof (harder)

Prove algebraically that the difference between the squares of any two odd numbers is a multiple of 8.

Let the odd numbers be 2a + 1 and 2b + 1, where a and b are integers.
$(2a + 1)^2 - (2b + 1)^2 = 4a^2 + 4a - 4b^2 - 4b = 4(a - b)(a + b + 1)$
$(a - b) + (a + b + 1) = 2a + 1$ is odd, so one of the two brackets is even
So $(a - b)(a + b + 1)$ is even and $4(a - b)(a + b + 1)$ is a multiple of 8
**Answer: Proved**