Day 277 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

80 deer in a forest are caught, marked and put back.
Later, a second sample of 60 deer is caught. 12 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 12 out of 60

This should match the proportion marked in the whole population: $\frac{12}{60} = \frac{80}{N}$

$N = 80 \times 60 \div 12 = 400$

(b) No deer are born, die or leave between the samples, and the deer mix evenly

Answer: (a) 400 (b) The population stays the same between samples

2 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 90°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: $360 - 90 - 90 - 90 = 90°$
(b) The angle at the circumference is half the angle at the centre: $90 \div 2 = 45°$
Answer: (a) 90° (b) 45°

3 Rationalising the denominator

Rationalise the denominator of $\frac{3}{4\sqrt{6}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{6}$:

$\frac{3}{4\sqrt{6}} = \frac{3\sqrt{6}}{24}$

$= \frac{\sqrt{6}}{8}$

Answer: $\frac{\sqrt{6}}{8}$

4 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 58°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 58°
Angle AOB = $180 - 2 \times 58 = 64°$
(b) The angle at the circumference is half the angle at the centre: $64 \div 2 = 32°$
Answer: (a) 64° (b) 32°

5 Average speed

Sofia drives 10 km at an average speed of 20 km/h.
The drive then continues for another 200 km at an average speed of 80 km/h.
Work out the average speed for the whole journey.

Time for first part = $10 \div 20 = 0.5$ hours
Time for second part = $200 \div 80 = 2.5$ hours
Total distance = 210 km, total time = 3 hours
Average speed = $210 \div 3 = 70$ km/h
Answer: 70 km/h

Day 277 **S Tier 10-a-day: Model Solutions**

6 Transformations of graphs

The point P(6, 6) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x + 4) (b) y = −f(x)

(a) y = f(x + 4) is a translation of 4 to the left: (2, 6)
(b) y = −f(x) is a reflection in the x-axis, so the y coordinate changes sign: (6, −6)
Answer: (a) (2, 6) (b) (6, −6)

7 Fractions of fractions

Dina read $\frac{2}{7}$ of a book on Saturday.

On Sunday she read $\frac{4}{5}$ of the pages that were left.

She read 40 pages on Sunday.
How many pages are in the book?

The second amount is $\frac{4}{5}$ of $\frac{5}{7}$, which is $\frac{4}{7}$ of the total

$\frac{4}{7}$ of the total is 40

Total = $40 \div \frac{4}{7} = 70$

Answer: 70 pages

8 Winning a match

Sofia and Priya play a match. The first to win 2 chess games wins the match.

The probability that Sofia wins any one of the chess games is $\frac{3}{10}$.

Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Sofia wins then loses) = $\frac{3}{10} \times \frac{7}{10} = \frac{21}{100}$

P(Sofia loses then wins) = $\frac{7}{10} \times \frac{3}{10} = \frac{21}{100}$

Total = $2 \times \frac{21}{100} = \frac{21}{50}$

Answer: $\mathbf{\frac{21}{50}}$

9 Rearranging when the subject appears twice

Make r the subject of $P = \frac{-5r + 1}{r + 2}$

Multiply by r + 2: $P(r + 2) = -5r + 1$
Expand: $rP + 2P = -5r + 1$
Collect the r terms: $rP + 5r = 1 - 2P$
Factorise: $r(P + 5) = -2P + 1$
$r = \frac{-2P + 1}{P + 5}$

Answer: $\mathbf{r = \frac{-2P + 1}{P + 5}}$

Day 277 **S Tier 10-a-day: Model Solutions**

10 Angles of elevation

A point A is on level ground in line with the base of a tower. (Calculator)
The angle of elevation of the top of the tower from A is 57°.
B is a point on the same line, 30 m further from the tower than A.
The angle of elevation of the top of the tower from B is 45°.
Work out the height of the tower. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 57° = \frac{h}{x}$, so $x = h \div \tan 57°$

From B: $\tan 45° = \frac{h}{x + 30}$, so $x + 30 = h \div \tan 45°$

Subtract: $30 = h \div \tan 45° - h \div \tan 57° = h(1.0000 - 0.64941)$
$h = 30 \div 0.35059 = 85.6$ m (1 d.p.)
Answer: 85.6 m