Day 276 **S Tier 10-a-day: Model Solutions**

1 Solving quadratics by factorising

Solve $9x^2 - 6x - 8 = 0$

Factorise: $(3x - 4)(3x + 2) = 0$

$3x - 4 = 0$ or $3x + 2 = 0$

$x = \frac{4}{3}$ or $x = -\frac{2}{3}$

Answer: $x = -\frac{2}{3}$ or $x = \frac{4}{3}$

2 Area of a triangle using sine

Triangle ABC has AB = 5 cm and BC = 16 cm.

The area of triangle ABC is $20\sqrt{2}$ cm^2.

Angle ABC is obtuse.

Work out the size of angle ABC.

Do not use a calculator.

Area $= \frac{1}{2} \times AB \times BC \times \sin B$

$20\sqrt{2} = \frac{1}{2} \times 5 \times 16 \times \sin B$

$\sin B = 20\sqrt{2} \div 40 = \frac{\sqrt{2}}{2}$

$\sin 45° = \frac{\sqrt{2}}{2}$, and B is obtuse, so $B = 180° - 45° = 135°$

Answer: 135°

3 Probabilities with an unknown

A biased wheel stops on one of four outcomes with these probabilities.

A: 3x; B: x + 0.2; C: 3x; D: 0.1

(a) Work out the value of x.

(b) The biased wheel is used 60 times. Work out an estimate for the number of times it stops on A.

(a) Probabilities add up to 1: $3x + x + 0.2 + 3x + 0.1 = 1$

$7x + 0.3 = 1$, so $7x = 0.7$

$x = 0.1$

(b) $P(A) = 3 \times 0.1 = 0.3$, so $60 \times 0.3 = 18$

Answer: (a) x = 0.1 (b) 18

4 LCM problems

A church bell rings every 6 minutes.

Another bell rings every 8 minutes.

A third bell rings every 15 minutes.

They all ring together at 10:00 am.

How many times do they all ring together between 10:00 am and 3:00 pm, including both 10:00 am and 3:00 pm if they do?

Find the LCM of 6, 8 and 15

$6 = 2 \times 3$, $8 = 2^3$, $15 = 3 \times 5$

LCM $= 2^3 \times 3 \times 5 = 120$

Total time from 10:00 am to 3:00 pm = 300 minutes

$300 \div 120 = 2.5$, so there are 2 more times after the start

Including the start: 2 + 1 = 3

Answer: 3

Day 276 **S Tier 10-a-day: Model Solutions**

5 Laws of indices with algebra

Simplify $(4x^4y^{-4})^{\frac{3}{2}}$

Raise each part to the power $\frac{3}{2}$

$4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$

$(x^4)^{\frac{3}{2}} = x^6$ and $(y^{-4})^{\frac{3}{2}} = y^{-6}$

$= 8x^6y^{-6}$

Answer: $8x^6y^{-6}$

6 Combined transformations

The point P(−1, −5) is rotated 180° about the origin.
The image is then reflected in the x-axis.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (1, 5)
After the second transformation: P'' = (1, −5)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, −y) → (−x, y)
The single mapping is (x, y) → (−x, y)
This is a reflection in the y-axis
Answer: (a) (1, −5) (b) Reflection in the y-axis

7 Bounds of area and perimeter

A rectangle has length 10 cm and width 16 cm.
Both measurements are correct to the nearest centimetre.
Chen says that the area of the rectangle could be exactly 180 cm^2.
Is Chen correct? You must show your working.

$9.5 \leq$ length < 10.5 and $15.5 \leq$ width < 16.5
Lower bound of area = 9.5 × 15.5 = 147.25
Upper bound of area = 10.5 × 16.5 = 173.25
So $147.25 \leq$ area < 173.25
180 is outside this interval, so Chen is not correct
Answer: No, 180 is outside the interval

8 Tangent to a circle

A circle has equation $(x + 1)^2 + (y - 1)^2 = 130$.
The point P(10, 4) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (−1, 1). The gradient of the radius to P is $\frac{4-1}{10-(-1)} = \frac{3}{11}$

The tangent is perpendicular to the radius, so its gradient is $-\frac{11}{3}$. Using P: $y - 4 = -\frac{11}{3}(x - 10)$

Multiply by 3: $3(y - 4) = -11(x - 10)$
Expand and rearrange: $11x + 3y = 122$
Answer: $11x + 3y = 122$

Day 276 **S Tier 10-a-day: Model Solutions**

9 Cone made from a sector

A sector of a circle has radius 24 cm and angle 180°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{180}{360} \times 2\pi \times 24 = 24\pi$ cm

This is the circumference of the base: $2\pi r = 24\pi$, so r = 12 cm

(b) Slant height = 24 cm, so height$^2 = 24^2 - 12^2 = 432$, height = $12\sqrt{3}$ cm

Volume = $\frac{1}{3} \times \pi \times 12^2 \times 12\sqrt{3} = 576\sqrt{3}\pi$ cm^3

Answer: (a) 12 cm (b) $576\sqrt{3}\pi$ cm^3

10 Ratio and money transferred

Callum and Maya share some money in the ratio 5 : 7.
Callum gives Maya £7.
The ratio of their money is now 3 : 5.
How much money do they have altogether?

Let the amounts be 5k and 7k, so the total is 12k

Callum now has 5k − 7, which is $\frac{3}{8}$ of the total

8(5k − 7) = 3 × 12k
40k − 56 = 36k, so 4k = 56 and k = 14
Total = 12 × 14 = £168
Answer: £168