**Day 275** **S Tier 10-a-day: Model Solutions**

## 1 Factorising quadratics

Factorise $3x^2 - 4x - 15$

$3 \times (-15) = -45$. Two numbers that multiply to $-45$ and add to $-4$ are 5 and $-9$

Split the middle term: $3x^2 + 5x - 9x - 15$

Factorise in pairs: $x(3x + 5) - 3(3x + 5)$

$= (x - 3)(3x + 5)$

**Answer: $(x - 3)(3x + 5)$**

## 2 Mean of a changing set

The mean of 9 numbers is 85.

A new number is added and the mean of all 10 numbers is 78.2.

Work out the new number.

Total of the 9 numbers $= 9 \times 85 = 765$

New total $= 10 \times 78.2 = 782$

New number $= 782 - 765 = 17$

**Answer: 17**

## 3 Simplifying surds

Write $\sqrt{50} - \sqrt{8} + \sqrt{2}$ in the form $a\sqrt{2}$

where a is an integer.

$\sqrt{50} = 5\sqrt{2}$; $\sqrt{8} = 2\sqrt{2}$

$(5 - 2 + 1)\sqrt{2} = 4\sqrt{2}$

**Answer: $4\sqrt{2}$**

## 4 Ratio and algebra

In an animal shelter, the ratio of the number of cats to the number of dogs is 2 : 5.

Then 20 more cats are added and 14 more dogs are added.

The ratio is now 8 : 11.

How many dogs were there at the start?

Let the numbers at the start be 2k and 5k

$(2k + 20) : (5k + 14) = 8 : 11$, so $11(2k + 20) = 8(5k + 14)$

$22k + 220 = 40k + 112$

$-18k = -108$, so $k = 6$

Number of dogs at the start: $5 \times 6 = 30$

**Answer: 30**

## 5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 6 cm, DB = 9 cm and the area of triangle ADE is 16 $cm^2$.

Work out the area of the trapezium DBCE.

The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 6 + 9 = 15 cm

Length scale factor $= 15 \div 6 = \frac{5}{2}$

Area scale factor $= (\frac{5}{2})^2 = \frac{25}{4}$

Area of triangle ABC $= 16 \times \frac{25}{4} = 100$ $cm^2$

Trapezium $= 100 - 16 = 84$ $cm^2$

**Answer: 84 $cm^2$**

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## 6 Equal roots and real roots

Show that the equation $x^2 + 3kx - 7 = 0$ has two distinct real solutions for all values of k.

$b^2 - 4ac = (3k)^2 - 4 \times 1 \times (-7)$

$= 9k^2 + 28$

$9k^2$ is never negative, so $9k^2 + 28$ is always at least 28, which is greater than 0

So $b^2 - 4ac > 0$ for all k, and there are always two distinct real solutions

**Answer: Shown**

## 7 Shaded areas: exact answers

The diagram shows an equilateral triangle inside a circle of radius 2 cm.
The three vertices of the triangle lie on the circle.
Work out the area of the shaded region. Give your answer in the form $a\pi - b\sqrt{3}$.

Split the triangle into 3 isosceles triangles with two sides of 2 cm and an angle of 120° between them

Area of the triangle = $3 \times \frac{1}{2} \times 2 \times 2 \times \sin 120° = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

Area of circle = $\pi \times 2^2 = 4\pi$

Shaded area = $4\pi - 3\sqrt{3}$ $cm^2$

**Answer: $(4\pi - 3\sqrt{3})$ $cm^2$**

## 8 Rationalising the denominator

Write $\frac{5 + \sqrt{3}}{6 - \sqrt{3}}$ in the form $p + q\sqrt{3}$
where p and q are rational numbers.

Multiply top and bottom by $(6 + \sqrt{3})$:

Numerator: $(5 + \sqrt{3})(6 + \sqrt{3}) = 33 + 11\sqrt{3}$

Denominator: $36 - 3 = 33$

$= \frac{3 + \sqrt{3}}{3}$

**Answer: $\frac{3 + \sqrt{3}}{3}$**

## 9 Geometric sequences

The 3rd term of a geometric sequence is 24.
The 5th term of the sequence is 96.
All the terms of the sequence are positive.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 96 \div 24 = 4$

$r = \sqrt{4} = 2$ (positive, since all terms are positive)

(b) The 3rd term is $a \times r^2$: $a \times 4 = 24$

$a = 24 \div 4 = 6$

**Answer: (a) r = 2 (b) 6**

## 10 Proof with indices

Show that $5^{k+2} + 5^k$ is always a multiple of 26.

$5^{k+2} = 5^k \times 5^2$

$5^{k+2} + 5^k = 5^k(5^2 + 1)$

$5^2 + 1 = 26$

$= 26 \times 5^k$, which is a multiple of 26

**Answer: $26 \times 5^k$**