**Day 274** **S Tier 10-a-day: Model Solutions**

## 1 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 29 is taken. Work out how many come from Year 9.
(b) A different stratified sample has 18 from Year 7. Work out the size of this sample.

Total = 90 + 80 + 90 + 30 = 290

(a) Year 9: $\frac{90}{290} \times 29 = 9$

(b) Fraction sampled = $18 \div 90 = \frac{1}{5}$, so sample size = $\frac{1}{5} \times 290 = 58$

**Answer: (a) 9 (b) 58**

## 2 Comparing costs

FitLife has a joining fee of £8 plus £4.50 per month.
GymNow has a joining fee of £3.50 plus £5 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 12 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when $8 + 4.50n = 3.50 + 5n$
$0.50n = 4.50$, so $n = 9$
(b) FitLife: £8 + £4.50 × 12 = £62
GymNow: £3.50 + £5 × 12 = £63.50
FitLife is cheaper by £1.50
**Answer: (a) 9 (b) FitLife, by £1.50**

## 3 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 50 cm. (Calculator)
The outer radius is 6 cm and the inner radius is 4 cm.
The density of the metal is 11.3 g/cm$^3$.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 6^2 \times 50 - \pi \times 4^2 \times 50 = \pi(36 - 16) \times 50$
$= 3141.6$ cm$^3$
Mass = density × volume = 11.3 × 3141.6 = 35500 g
= 35.5 kg (3 s.f.)
**Answer: 35.5 kg**

## 4 Completing the square

(a) Write $x^2 - 10x + 21$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 - 10x + 21$

(a) Half of −10 is −5, so $x^2 - 10x = (x - 5)^2 - 25$
$x^2 - 10x + 21 = (x - 5)^2 - 25 + 21$
$= (x - 5)^2 - 4$
(b) The minimum is when $x - 5 = 0$, so $x = 5$ and $y = -4$
**Answer: (a) $(x - 5)^2 - 4$ (b) (5, −4)**

## 5 Rationalising the denominator

Rationalise the denominator of $\frac{2}{3\sqrt{7}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{7}$:
$\frac{2}{3\sqrt{7}} = \frac{2\sqrt{7}}{21}$

**Answer: $\frac{2\sqrt{7}}{21}$**

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## 6 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 2) cm, DE = 9 cm and BC = 21 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 2} = \frac{9}{21}$
$21x = 9(2x + 2)$
$21x = 18x + 18$
$3x = 18$
$x = 6$
(b) $AB = 2x + 2 = 14$ cm
**Answer: (a) x = 6 (b) 14 cm**

## 7 Recurring decimals

Without using a calculator, work out $0.\dot{9}\dot{4} + 0.\dot{7}\dot{5}$
Give your answer as a fraction in its simplest form.

$0.\dot{9}\dot{4} = \frac{94}{99}$

$0.\dot{7}\dot{5} = \frac{25}{33}$

$\frac{94}{99} + \frac{25}{33} = \frac{169}{99}$

**Answer:** $\mathbf{\frac{169}{99}}$

## 8 Estimating the mean from a histogram table

The table gives information about the times of some people.
Work out an estimate for the mean time.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 7, 5, 34, 28
Mid-points: 5, 15, 30, 50
$\Sigma fm = 7\times5 + 5\times15 + 34\times30 + 28\times50 = 2530$, $\Sigma f = 74$
Mean = 2530 ÷ 74 = 34.2
**Answer: 34.2**

## 9 Standard form

Work out $(4.9 \times 10^7) + (5.1 \times 10^4)$
Give your answer in standard form.

Write the first number as a multiple of $10^4$: $4.9 \times 10^7 = 4900 \times 10^4$
$(4900 + 5.1) \times 10^4 = 4905.1 \times 10^4$
$= 4.9051 \times 10^7$
**Answer: $4.9051 \times 10^7$**

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## 10 Tangency conditions

The line $y = 2x + k$ is a tangent to the circle $x^2 + y^2 = 19$.
Find the possible values of k.
Give any surds in their simplest form.

Substitute $y = 2x + k$: $x^2 + (2x + k)^2 = 19$
Expand: $5x^2 + 4kx + k^2 - 19 = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $(4k)^2 - 4 \times 5 \times (k^2 - 19) = 0$
$-4k^2 + 380 = 0$, so $k^2 = 95$
$k = \pm\sqrt{95}$
**Answer: $k = \pm\sqrt{95}$**