Day 273 **S Tier 10-a-day: Model Solutions**

1 Transformations of graphs

The graph of y = f(x) has a maximum point at (3, 1).
Write down the coordinates of the turning point of the graph of
(a) y = f(x + 3)
(b) y = f(x − 2) + 1

(a) Start with the turning point (3, 1)
Apply the transformation to the point: (0, 1)
(b) f(x − 2) changes the x-coordinate to 5, and f(x) + 1 changes the y-coordinate to 2
Turning point: (5, 2)
Answer: (a) (0, 1) (b) (5, 2)

2 Index equations

Solve $8^x = 16^3$

Write both sides as powers of 2: $8 = 2^3$ and $16 = 2^4$
Equate the powers: 3x = 12
3x = 12, so x = 4
Answer: x = 4

3 Area of a segment

The circle has centre O and radius 11 cm. A and B are points on the circle and angle AOB = 100°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{100}{360} \times \pi \times 11^2 = 105.59$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 11 \times 11 \times \sin 100° = 59.581$ cm^2

Segment = sector − triangle = 105.59 − 59.581 = 46.012
= 46.0 cm^2 (3 s.f.)
Answer: 46.0 cm^2

4 Simultaneous equations

Solve the simultaneous equations
5x + 6y = −34
6x − 4y = 4

To match the x terms: (1) × 6 gives 30x + 36y = −204, and (2) × 5 gives 30x − 20y = 20
Subtract the equations to eliminate x: 56y = −224
y = −4
Substitute y = −4 into (1): 5x − 24 = −34
5x = −10, so x = −2
Answer: x = −2, y = −4

5 Compound interest

Amir invests £1,000 for 4 years at 4.5% per year compound interest.
Work out the value of the investment at the end of the 4 years.
Give your answer to the nearest penny. (Calculator)

Multiplier = 1 + 4.5 ÷ 100 = 1.045
$1000 \times 1.045^4 = 1192.52$
To the nearest penny: £1,192.52
Answer: £1,192.52

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6 Laws of indices

Write $\frac{16^{n+1} \times 4^{n-3}}{2^n}$ in the form 2^{an+c}

where a and c are integers.

$16 = 2^4$, $4 = 2^2$, $2 = 2^1$

$16^{n+1} = 2^{4(n+1)}$, $4^{n-3} = 2^{2(n-3)}$, $2^n = 2^n$

Exponent: $4(n + 1) + 2(n - 3) - n = 5n - 2$

Answer: a = 5, c = −2

7 Similar triangles with algebra

In the diagram, DE is parallel to BC.

AD = x cm, DB = (x + 3) cm, DE = 9 cm and BC = 21 cm.

(a) Find the value of x.

(b) Work out the length of AB.

The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x+3} = \frac{9}{21}$

$21x = 9(2x + 3)$

$21x = 18x + 27$

$3x = 27$

$x = 9$

(b) AB = 2x + 3 = 21 cm

Answer: (a) x = 9 (b) 21 cm

8 Bounds of speed, density and pressure

A runner covers a distance of 200 m, correct to 1 significant figure.

The race takes 11 s, correct to 2 significant figures.

Work out the lower bound of the speed.

Give your answer correct to 3 significant figures. (Calculator)

$150 \leq$ distance < 250; $10.5 \leq$ time < 11.5

Lower bound of speed = smallest distance ÷ biggest time = 150 ÷ 11.5

= 13.0435... = 13.0 (3 s.f.)

Answer: 13.0 m/s

9 Terms of a quadratic sequence

The nth term of a sequence is $2n^2 + 6n - 8$.

(a) Work out the 10th term.

(b) Is 411 a term of the sequence? You must show your working.

(a) $2 \times 10^2 + 6 \times 10 - 8 = 252$

(b) Solve $2n^2 + 6n - 8 = 411$, so $2n^2 + 6n - 419 = 0$

$b^2 - 4ac = 6^2 - 4 \times 2 \times (-419) = 3388$, which is not a square number

So n is not a whole number, and 411 is not a term

Answer: (a) 252 (b) No

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10 Ratio and money transferred

Leila and Carlos share some money in the ratio 5 : 2.
Leila gives Carlos £39.
The ratio of their money is now 2 : 5.
How much money do they have altogether?

Let the amounts be 5k and 2k, so the total is 7k

Leila now has $5k - 39$, which is $\frac{2}{7}$ of the total

$7(5k - 39) = 2 \times 7k$
$35k - 273 = 14k$, so $21k = 273$ and $k = 13$
Total $= 7 \times 13 = £91$
Answer: £91