Day 272 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(2x + 30)°$, $(2x + 40)°$, $(x + 15)°$, $(2x - 25)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$8x + 60 = 540$, so $8x = 480$
$x = 60$
(b) The angles are $60°$, $150°$, $160°$, $75°$, $95°$
The largest is $160°$
Answer: (a) x = 60 (b) 160°

2 Factorising quadratics

Factorise $6x^2 - 35x + 49$

$6 \times 49 = 294$. Two numbers that multiply to 294 and add to -35 are -21 and -14
Split the middle term: $6x^2 - 21x - 14x + 49$
Factorise in pairs: $3x(2x - 7) - 7(2x - 7)$
$= (3x - 7)(2x - 7)$
Answer: (3x − 7)(2x − 7)

3 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(5x + 15)°$
Angle PQR = $(2x + 10)°$
Angle PRS = $(5x + 45)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$5x + 45 = 7x + 25$
$2x = 20$
$x = 10$
(b) Angle PRS = $95°$ so angle PRQ = $180° - 95° = 85°$
Answer: (a) x = 10 (b) 85°

4 Speed-time graphs

The graph shows the speed of a car during 24 seconds.
(a) Work out the acceleration during the first 6 seconds.
(b) Work out the average speed over the whole 24 seconds.

(a) Acceleration = change in speed ÷ time = $(24 - 0) \div 6 = 4$ m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{0 + 24}{2} \times 6 = 72$; rectangle: $24 \times 11 = 264$; triangle: $\frac{1}{2} \times 7 \times 24 = 84$
Total = 420 m, so average speed = $420 \div 24 = 17.5$ m/s
Answer: (a) 4 m/s^2 (b) 17.5 m/s

5 Percentages

C is 50% less than P.
P is 60% less than A.
Work out C as a percentage of A.

$C = 0.5 \times P$ and $P = 0.4 \times A$
$C = 0.5 \times 0.4 \times A = 0.2 \times A$
$0.2 \times 100 = 20\%$
Answer: 20%

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6 Quadratic sequences

Here are the first five terms of a quadratic sequence.
$-6, -1, 8, 21, 38$
Find an expression for the nth term.

First differences: 5, 9, 13, 17
Second differences: 4, 4, 4, so a = 4 ÷ 2 = 2
Subtract $2n^2$ from each term: $-8, -9, -10, -11, -12$
The remaining linear sequence has nth term $-n - 7$
nth term = $2n^2 - n - 7$
Answer: $2n^2 - n - 7$

7 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $5\mathbf{b}$, OQ = $-3\mathbf{a} + 2\mathbf{b}$ and OR = $-5\mathbf{a}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-3\mathbf{a} - 3\mathbf{b}$
QR = OR − OQ = $-2\mathbf{a} - 2\mathbf{b}$
QR = $\frac{2}{3}$ PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 2
Answer: (b) 3 : 2

8 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 5$
$x + y = 1$

From the second equation, $y = -x + 1$
Substitute into the first equation: $x^2 + (-x + 1)^2 = 5$
Expand: $2x^2 - 2x - 4 = 0$
Divide by 2: $x^2 - x - 2 = 0$
Factorise: $(x + 1)(x - 2) = 0$, so $x = -1$ or $x = 2$
$y = -x + 1$ gives $y = 2$ and $y = -1$
Answer: $x = -1$, $y = 2$ and $x = 2$, $y = -1$

9 HCF and LCM

The HCF of two numbers is 5 and their LCM is 30.
The smaller number is 10.
Work out the larger number.

HCF × LCM = product of the two numbers
5 × 30 = 150
Larger number = 150 ÷ 10 = 15
Answer: 15

10 Moving averages

The table shows the daily visitors over six periods.
The 3-point moving average centred on Thu is 23.
(a) Work out the value of x.
(b) Work out the 3-point moving average centred on Fri.

(a) (34 + x + 24) ÷ 3 = 23
34 + x + 24 = 69, so x = 11
(b) (11 + 24 + 13) ÷ 3 = 48 ÷ 3 = 16
Answer: (a) x = 11 (b) 16