Day 271 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $xyz - x^2$
when $x = -1$, $y = -1$ and $z = -2$.

Substitute the values: $(-1) \times (-1) \times (-2) - (-1)^2$
$= -2 - 1$
$= -3$
Answer: -3

2 Capture and recapture

60 beetles in a field are caught, marked and put back.
Later, a second sample of 100 beetles is caught. 4 of these are marked.
(a) Work out an estimate for the number of beetles in a field.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 4 out of 100

This should match the proportion marked in the whole population: $\frac{4}{100} = \frac{60}{N}$

$N = 60 \times 100 \div 4 = 1500$
(b) No beetles are born, die or leave between the samples, and the beetles mix evenly
Answer: (a) 1500 (b) The population stays the same between samples

3 Equations with fractions

Solve $\frac{x-4}{6} + \frac{x-4}{2} = -2$

Multiply every term by 6: $(x - 4) + 3(x - 4) = -12$
Expand: $4x - 16 = -12$
$4x = 4$
$x = 1$
Answer: $x = 1$

4 Enlargement

Triangle P has vertices A(0, 1), B(−1, 3) and C(1, 2).
Triangle Q has vertices A'(3, −2), B'(5, −6) and C'(1, −4).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = −1, x-step from A' to B' = 2
Scale factor = 2 ÷ −1 = −2
The centre C satisfies: C + −2(A − C) = A', so (1 − −2)C = A' − −2 A
x: 3 × x = 3 − (−2) × 0 = 3, so x = 1
y: 3 × y = (−2) − (−2) × 1 = 0, so y = 0
Answer: Enlargement, scale factor −2, centre (1, 0)

5 Speed-time graphs

The graph shows the speed of a cyclist during a journey.
(a) Work out the deceleration in the last 12 seconds.
(b) Work out the average speed over the whole journey.

(a) Deceleration = 24 ÷ 12 = 2 m/s^2

(b) Total distance = area under the graph = $\frac{1}{2} \times 8 \times 24 + 24 \times 10 + \frac{1}{2} \times 12 \times 24 = 480$ m

Average speed = total distance ÷ total time = 480 ÷ 30 = 16 m/s
Answer: (a) 2 m/s^2 (b) 16 m/s

Day 271 **S Tier 10-a-day: Model Solutions**

6 Standard form problems

Light travels at 3×10^8 metres per second.
A planet is 9×10^{11} metres from a spacecraft.
How many seconds does a light signal take to travel from the planet to the spacecraft?
Give your answer in standard form.

Time = distance ÷ speed
$(9 \times 10^{11}) \div (3 \times 10^8)$
$9 \div 3 = 3$ and $10^{11} \div 10^8 = 10^3$
$= 3 \times 10^3$
Answer: 3×10^3 seconds

7 Bearings with the sine and cosine rules

A walker leaves a camp P and travels 100 km on a bearing of 265° to Q. (Calculator)
It then travels 44 km on a bearing of 315° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 265° − 180° = 085°
Angle PQR = 130° (the difference between the bearings 085° and 315°)
$PR^2 = 100^2 + 44^2 - 2 \times 100 \times 44 \times \cos 130° = 17593$, so PR = 132.6 km
(b) $\cos(QPR) = \frac{100^2 + PR^2 - 44^2}{2 \times 100 \times PR}$, so angle QPR = 14.7°
R is clockwise of the direction PQ, so the bearing is 265° + 15° = 280°
Answer: (a) 132.6 km (b) 280°

8 Finding a percentage rate

Tom invests £1,000 in an account paying compound interest.
The interest rate is the same percentage each year.
After 3 years the account holds £3,375.
Work out the annual percentage interest rate.

Multiplier for 3 years = £3,375 ÷ £1,000 = 3.375
Yearly multiplier = the cube root of 3.375 = 1.5
50% increase each year
Answer: 50%

9 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 40.
Work out the value of x.

Mid-points: 10, 30, 50, 70
Estimated mean = $\Sigma fm \div \Sigma f$, so $(1770 + 50x) \div (45 + x) = 40$
$1770 + 50x = 40(45 + x) = 1800 + 40x$
$(50 - 40)x = 1800 - 1770$
$10x = 30$, so $x = 3$
Answer: $x = 3$

10 Squares, cubes and prime factors

Find the smallest positive integer k such that 12,348 × k is a cube number.

$12{,}348 = 2^2 \times 3^2 \times 7^3$
For a cube number every power must be a multiple of 3
Multiply by 2×3 to get $2^3 \times 3^3 \times 7^3$
k = 6
Answer: $k = 6$