Day 270 **S Tier 10-a-day: Model Solutions**

1 Recurring decimals

Convert the recurring decimal $0.\dot{0}7\dot{6}$ to a fraction.
Give your fraction in its simplest form.

Let x = 0.076076...
1000x = 76.076076...
Subtract: 999x = 76
$x = \frac{76}{999}$

Answer: $\frac{76}{999}$

2 Capture and recapture

50 deer in a forest are caught, marked and put back.
Later, a second sample of 60 deer is caught. 12 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 12 out of 60
This should match the proportion marked in the whole population: $\frac{12}{60} = \frac{50}{N}$
N = 50 × 60 ÷ 12 = 250
(b) No deer are born, die or leave between the samples, and the deer mix evenly
Answer: (a) 250 (b) The population stays the same between samples

3 Completing the square

(a) Write $3x^2 - 36x + 116$ in the form $a(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = 3x^2 - 36x + 116$

(a) Take out the factor 3 from the x terms: $3(x^2 - 12x) + 116$
$x^2 - 12x = (x - 6)^2 - 36$, so $3x^2 - 36x + 116 = 3[(x - 6)^2 - 36] + 116$
$= 3(x - 6)^2 + 8$
(b) The minimum is when $x - 6 = 0$, so x = 6 and y = 8
Answer: (a) $3(x - 6)^2 + 8$ (b) (6, 8)

4 Expanding brackets with surds

Expand and simplify $(5 + 2\sqrt{2})^2$
Give your answer in the form $p + q\sqrt{2}$

$(5 + 2\sqrt{2})^2 = 25 + 20\sqrt{2} + 8$
$= 33 + 20\sqrt{2}$
Answer: $33 + 20\sqrt{2}$

5 Quadratic inequalities

Solve $2x^2 + 16x + 30 > 0$

Divide by 2: $x^2 + 8x + 15 > 0$
Solve $x^2 + 8x + 15 = 0$: $(x + 5)(x + 3) = 0$, so $x = -5$ or $x = -3$
The graph is a U shape crossing the x-axis at -5 and -3; it is above the axis outside the roots
$x < -5$ or $x > -3$
Answer: $x < -5$ or $x > -3$

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6 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the opposite side BC and the known side is the adjacent side AB, so use tan 45°
tan 45° = 1
$x = 4\sqrt{2} \times \tan 45° = 4\sqrt{2} \times 1$
$x = 4\sqrt{2}$
Answer: $x = 4\sqrt{2}$

7 Exponential growth and decay

A population of members of a website starts at 3,000 and grows by 25% each year.
After how many whole years will the population first be at least double its starting value? (Calculator)

Multiplier = 1.25, double is 6,000
After 3 years: $3000 \times 1.25^3 = 5859.4$
After 4 years: $3000 \times 1.25^4 = 7324.2$
Answer: 4 years

8 Laws of indices

Write $\frac{16^{n+2} \times 4^{n+1}}{2^n}$ in the form 2^{an+c}
where a and c are integers.

$16 = 2^4$, $4 = 2^2$, $2 = 2^1$
$16^{n+2} = 2^{4(n+2)}$, $4^{n+1} = 2^{2(n+1)}$, $2^n = 2^n$
Exponent: $4(n + 2) + 2(n + 1) - n = 5n + 10$
Answer: a = 5, c = 10

9 Angles of elevation

A point A is on level ground in line with the base of a lighthouse. (Calculator)
The angle of elevation of the top of the lighthouse from A is 56°.
B is a point on the same line, 50 m further from the lighthouse than A.
The angle of elevation of the top of the lighthouse from B is 44°.
Work out the height of the lighthouse. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 56° = \frac{h}{x}$, so $x = h \div \tan 56°$

From B: $\tan 44° = \frac{h}{x + 50}$, so $x + 50 = h \div \tan 44°$
Subtract: $50 = h \div \tan 44° - h \div \tan 56° = h(1.0355 - 0.67451)$
$h = 50 \div 0.36102 = 138.5$ m (1 d.p.)
Answer: 138.5 m

10 Ratio and expressions

2x = 3y
Work out the ratio (6x − 3y) : (5x + 4y)
Give your answer in its simplest form.

2x = 3y gives x : y = 3 : 2, so let x = 3t and y = 2t
6(3t) − 3(2t) = 12t
5(3t) + 4(2t) = 23t
12t : 23t = 12 : 23
Answer: 12 : 23