Day 27 **S Tier 10-a-day: Model Solutions**

1 Comparing costs

Zip Taxis has a fixed charge of £6.50 plus £4.50 per mile.
Go Cabs has a fixed charge of £51.50 plus £1.50 per mile.
(a) Work out the number of miles for which the two costs are the same.
(b) Which is cheaper for 21 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when $6.50 + 4.50n = 51.50 + 1.50n$
$3n = 45$, so $n = 15$
(b) Zip Taxis: £6.50 + £4.50 × 21 = £101
Go Cabs: £51.50 + £1.50 × 21 = £83
Go Cabs is cheaper by £18
Answer: (a) 15 (b) Go Cabs, by £18

2 Percentage change

A 5% increase followed by a 15% increase is equivalent to a single percentage change.
Work out the single percentage change that has the same effect.

Multipliers: 1.05 and 1.15
Overall multiplier = 1.05 × 1.15 = 1.2075
1.2075 − 1 = 0.2075, so an overall increase of 20.75%
Answer: 20.75% increase

3 Mean of a changing set

The mean of 6 numbers is 84.5.
One number is removed and the mean of the remaining 5 numbers is 91.4.
Work out the number that was removed.

Total of the 6 numbers = 6 × 84.5 = 507
Total of the 5 remaining = 5 × 91.4 = 457
Number removed = 507 − 457 = 50
Answer: 50

4 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (4x)°, angle BCD = (2x + 6)° and angle ABC = (3x + 30)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
(4x) + (2x + 6) = 180
6x + 6 = 180, so 6x = 174
x = 29
(b) Angle ABC = 3 × 29 + 30 = 117°
Angle ADC = 180 − 117 = 63°
Answer: (a) x = 29 (b) 63°

5 Expanding three brackets

Expand and simplify $(x + 3)^2(x - 1)$

Expand the first two brackets: $(x + 3)^2 = x^2 + 6x + 9$
Multiply by $(x - 1)$: $x(x^2 + 6x + 9) - (x^2 + 6x + 9)$
$= x^3 + 5x^2 + 3x - 9$
Answer: $x^3 + 5x^2 + 3x - 9$

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6 Prime factors and indices

Write $30^4 \times 12^2$ as a product of powers of prime numbers.

$30 = 2 \times 3 \times 5$, $12 = 2^2 \times 3$

$30^4 = 2^4 \times 3^4 \times 5^4$

$12^2 = 2^4 \times 3^2$

Add the powers of each prime: $2^8 \times 3^6 \times 5^4$

Answer: $2^8 \times 3^6 \times 5^4$

7 Ratio and mixtures

Container A holds juice and water in the ratio 6 : 5.
Container B holds juice and water in the ratio 3 : 6.
22 litres from container A and 18 litres from container B are mixed.
Work out the ratio of juice to water in the mixture.

From A: juice = $22 \times \frac{6}{11}$ = 12 litres, water = 10 litres

From B: juice = $18 \times \frac{1}{3}$ = 6 litres, water = 12 litres

juice: 12 + 6 = 18 and water: 10 + 12 = 22

18 : 22 = 9 : 11

Answer: 9 : 11

8 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 9 cm, top radius 2 cm and base radius 8 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+9} = \frac{2}{8}$

8 h = 2(h + 9), so 6 h = 18 and h = 3

Large cone: $\frac{1}{3} \times \pi \times 8^2 \times 12 = 256\pi$

Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 3 = 4\pi$

Frustum = $256\pi - 4\pi = 252\pi$ cm^3

Answer: 252π cm^3

9 Rearranging when the subject appears twice

Make t the subject of $T = \frac{7t+5}{t-6}$

Multiply by $t - 6$: $T(t - 6) = 7t + 5$
Expand: $tT - 6T = 7t + 5$
Collect the t terms: $tT - 7t = 6T + 5$
Factorise: $t(T - 7) = 6T + 5$
$t = \frac{6T+5}{T-7}$

Answer: $t = \frac{6T+5}{T-7}$

10 Profit and loss

A shop buys a games console and adds a 30% mark-up to get the selling price.
In a sale the selling price is reduced by 10% to £304.20.
Work out how much the shop paid for a games console.

Overall multiplier = 1.3 × 0.9 = 1.17
Cost = £304.20 ÷ 1.17
= £260

Answer: £260