Day 269 **S Tier 10-a-day: Model Solutions**

1 Exchange rates

Jamal changes £1100 into dollars at £1 = $1.15.
Jamal spends $70 and then changes the rest back into pounds at £1 = $1.25.
How many pounds does Jamal get back?

£1100 × 1.15 = $1265
1265 − 70 = $1195
1195 ÷ 1.25 = £956
Answer: £956

2 Substitution

Work out the value of $\sqrt{n^2 - 4mk}$
when m = 3, n = −2 and k = −1.

Substitute the values: $\sqrt{(-2)^2 - 4 \times 3 \times (-1)}$
$= \sqrt{16}$
= 4
Answer: 4

3 Exact trigonometric values

Work out the exact value of tan 60° + cos 45°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\tan 60° + \cos 45° = \sqrt{3} + \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{23}}{2}$

Answer: $\frac{\sqrt{23}}{2}$

4 Multiplying and dividing surds

Simplify fully $\frac{2\sqrt{24}}{\sqrt{6}}$

$\sqrt{24} = 2\sqrt{6}$
$\frac{2\sqrt{24}}{\sqrt{6}} = \frac{4\sqrt{6}}{\sqrt{6}}$
= 4
Answer: 4

5 Using a line of best fit

The scatter graph shows hours of revision (x) and test mark (y).
The equation of the line of best fit is y = 2.5x + 11.
(a) Estimate y when x = 5.
(b) Explain why an estimate of y when x = 25 may be unreliable.

(a) y = 2.5 × 5 + 11 = 23.5
(b) x = 25 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 23.5 (b) x = 25 is outside the range of the data

Day 269 **S Tier 10-a-day: Model Solutions**

6 Compound interest

Jess puts £1,000 in a savings account.
Compound interest is paid at 6% in year 1, 5% in year 2 and 1% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.06, 1.05, 1.01
£1,000 × 1.06 × 1.05 × 1.01
= £1,124.13
Answer: £1,124.13

7 Volume and surface area of composite solids

A solid is made from a cone of height 7 cm and a hemisphere. Both have radius 8 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cone: $\frac{1}{3} \times \pi \times 8^2 \times 7 = \frac{448}{3}\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 8^3 = \frac{1024}{3}\pi$

Total = $\frac{448}{3}\pi + \frac{1024}{3}\pi = \frac{1472}{3}\pi$ cm^3

Answer: $\mathbf{\frac{1472}{3}\pi}$ **cm^3**

8 Algebraic proof (harder)

Prove algebraically that $k^3 + 5k$ is a multiple of 6 for all integer values of k.

$k^3 + 5k = (k^3 - k) + 6k = (k - 1)k(k + 1) + 6k$
$(k - 1)k(k + 1)$ is the product of three consecutive integers, so it contains a multiple of 2 and a multiple of 3, so it is a multiple of 6
$6k = 6 \times 1k$ is also a multiple of 6
The sum of two multiples of 6 is a multiple of 6
Answer: Proved

9 Recurring decimals

Without using a calculator, work out $0.\dot{5}\dot{6} - 0.\dot{1}$
Give your answer as a fraction in its simplest form.

$0.\dot{5}\dot{6} = \frac{56}{99}$

$0.\dot{1} = \frac{1}{9}$

$\frac{56}{99} - \frac{1}{9} = \frac{5}{11}$

Answer: $\mathbf{\frac{5}{11}}$

10 Tangency conditions

The line $y = x + k$ is a tangent to the circle $x^2 + y^2 = 4$.
Find the possible values of k.
Give any surds in their simplest form.

Substitute $y = x + k$: $x^2 + (x + k)^2 = 4$
Expand: $2x^2 + 2kx + k^2 - 4 = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $(2k)^2 - 4 \times 2 \times (k^2 - 4) = 0$
$-4k^2 + 32 = 0$, so $k^2 = 8$
$k = \pm 2\sqrt{2}$
Answer: $\mathbf{k = \pm 2\sqrt{2}}$