Day 268 **S Tier 10-a-day: Model Solutions**

1 Exchange rates

Nadia changes £400 into euros at £1 = €1.18.
Nadia spends €50 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Nadia get back?

£400 × 1.18 = €472
472 − 50 = €422
422 ÷ 1.6 = £263.75
Answer: £263.75

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (4x + 15)°
Angle PQR = (3x − 20)°
Angle PRS = (6x + 17)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
6x + 17 = 7x − 5
x = 22
(b) Angle PRS = 149° so angle PRQ = 180° − 149° = 31°
Answer: (a) x = 22 (b) 31°

3 Comparing costs

Hire Shop A has a delivery charge of £3.50 plus £6 per day.
Hire Shop B has a delivery charge of £27.50 plus £2 per day.
(a) Work out the number of days for which the two costs are the same.
(b) Which is cheaper for 11 days, and by how much?

(a) Let n be the number of days. Working in pounds, the costs are equal when 3.50 + 6n = 27.50 + 2n
4n = 24, so n = 6
(b) Hire Shop A: £3.50 + £6 × 11 = £69.50
Hire Shop B: £27.50 + £2 × 11 = £49.50
Hire Shop B is cheaper by £20
Answer: (a) 6 (b) Hire Shop B, by £20

4 Inverse proportion in context

12 workers can build a wall in 30 days.
After 6 days, 6 of the workers leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 12 × 30 = 360 worker-days
Work done = 12 × 6 = 72, so work left = 288
Workers left = 12 − 6 = 6
Time = 288 ÷ 6 = 48 days
Answer: 48 days

5 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 53°} = \frac{6}{\sin 54°}$

$x = \frac{6 \times \sin 53°}{\sin 54°} = 5.9230$

x = 5.9 cm (1 d.p.)
Answer: AC = 5.9 cm

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6 Percentages

R is 15% less than D.
D is 10% more than C.
Work out R as a percentage of C.

R = 0.85 × D and D = 1.1 × C
R = 0.85 × 1.1 × C = 0.935 × C
0.935 × 100 = 93.5%
Answer: 93.5%

7 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 45$
$y = x + 9$

Substitute into the first equation: $x^2 + (x + 9)^2 = 45$
Expand: $2x^2 + 18x + 36 = 0$
Divide by 2: $x^2 + 9x + 18 = 0$
Factorise: $(x + 6)(x + 3) = 0$, so $x = -6$ or $x = -3$
$y = x + 9$ gives $y = 3$ and $y = 6$
Answer: $x = -6$, $y = 3$ and $x = -3$, $y = 6$

8 Counting factors

How many factors does 160 have?
Include 1 and 160.

$160 = 2^5 \times 5$
Add 1 to each power and multiply: $6 \times 2 = 12$
Answer: 12

9 Direct and inverse proportion

P is directly proportional to the square root of t.
When t = 16, P = 20.
(a) Find a formula for P in terms of t.
(b) Find the value of P when t = 9.

(a) $P = k\sqrt{t}$
Substitute t = 16 and P = 20: $20 = k \times \sqrt{16}$
k = 5
$P = 5\sqrt{t}$
(b) $P = 5\sqrt{9} = 15$
Answer: (a) $P = 5\sqrt{t}$ (b) 15

10 Laws of indices

Write $\frac{9^{n+2} \times 3^{n-3}}{3^n}$ in the form 3^{an+c}
where a and c are integers.

$9 = 3^2$, $3 = 3^1$, $3 = 3^1$
$9^{n+2} = 3^{2(n+2)}$, $3^{n-3} = 3^{n-3}$, $3^n = 3^n$
Exponent: $2(n + 2) + (n - 3) - n = 2n + 1$
Answer: $a = 2$, $c = 1$