**Day 267** **S Tier 10-a-day: Model Solutions**

## 1 Equations with indices

Solve $9^{x-3} = 27^{x+3}$

Write both sides as powers of 3: $(3^2)^{x-3} = (3^3)^{x+3}$

$3^{2x-6} = 3^{3x+9}$

Equate the powers: $2x - 6 = 3x + 9$

$-x = 15$

$x = -15$

**Answer: $x = -15$**

## 2 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.

(a) Work out the total length of the boundary of the track.

(b) Work out the area enclosed by the track. Give your answers in terms of $\pi$.

The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 25: circumference $= 2 \times \pi \times 25 = 50\pi$

The two straight sides: $2 \times 80 = 160$

Boundary $= (160 + 50\pi)$ m

(b) The rectangle is 80 m by $2 \times 25 = 50$ m, so its area is $80 \times 50 = 4000$. Circle: $\pi \times 25^2 = 625\pi$

Area $= (4000 + 625\pi)$ m$^2$

**Answer: (a) $(160 + 50\pi)$ m (b) $(4000 + 625\pi)$ m$^2$**

## 3 Percentages

P is 10% less than Q.

Q is 60% more than A.

Work out P as a percentage of A.

$P = 0.9 \times Q$ and $Q = 1.6 \times A$

$P = 0.9 \times 1.6 \times A = 1.44 \times A$

$1.44 \times 100 = 144\%$

**Answer: 144%**

## 4 Speed and a moving train

A train travels at a constant speed of 108 km/h.

It takes 30 seconds to go completely through a tunnel which is 830 m long.

Work out the length of the train, in metres.

Speed = 108 km/h = $108{,}000 \div 3600 = 30$ m/s

Distance travelled in 30 s $= 30 \times 30 = 900$ m

This is the length of the tunnel plus the length of the train

Train $= 900 - 830 = 70$ m

**Answer: 70 m**

## 5 Area of a triangle using sine

Triangle ABC has AB = 19 cm and BC = 8 cm.

The area of triangle ABC is 38 cm$^2$.

Angle ABC is obtuse.

Work out the size of angle ABC.

Do not use a calculator.

Area $= \frac{1}{2} \times AB \times BC \times \sin B$

$38 = \frac{1}{2} \times 19 \times 8 \times \sin B$

$\sin B = 38 \div 76 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is obtuse, so $B = 180° - 30° = 150°$

**Answer: 150°**

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## 6 Iteration

Use the iteration formula $x_{n+1} = \sqrt[3]{4x_n + 1}$ with $x_0 = 4$ (Calculator)
to work out $x_1$, $x_2$ and $x_3$.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt[3]{4 \times 4 + 1} = 2.57128...$

$x_2 = \sqrt[3]{4 \times x_1 + 1} = 2.24303...$

$x_3 = \sqrt[3]{4 \times x_2 + 1} = 2.15243...$

Rounded: $x_1 = 2.571$, $x_2 = 2.243$, $x_3 = 2.152$

**Answer: $x_1 = 2.571$, $x_2 = 2.243$, $x_3 = 2.152$**

## 7 Using a line of best fit

The scatter graph shows distance from town (x) and price (y).
The equation of the line of best fit is $y = -0.5x + 39$.
(a) Estimate y when x = 3.
(b) Explain why an estimate of y when x = 25 may be unreliable.

(a) $y = -0.5 \times 3 + 39 = 37.5$
(b) x = 25 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)

**Answer: (a) 37.5 (b) x = 25 is outside the range of the data**

## 8 Transformations of graphs

The graph of y = f(x) has a minimum point at (5, −2).
Write down the coordinates of the turning point of the graph of
(a) y = f(x) − 2
(b) y = 2f(x + 3)

(a) Start with the turning point (5, −2)
Apply the transformation to the point: (5, −4)
(b) f(x + 3) changes the x-coordinate to 2, and 2f(x) changes the y-coordinate to −4
Turning point: (2, −4)

**Answer: (a) (5, −4) (b) (2, −4)**

## 9 Equation of a circle

A circle has centre C(−5, −2) and passes through the point P(−2, 0).
Find an equation of the circle.

$r^2 = CP^2 = (-2 - (-5))^2 + (0 - (-2))^2$
$= 9 + 4 = 13$
$(x + 5)^2 + (y + 2)^2 = 13$

**Answer: $(x + 5)^2 + (y + 2)^2 = 13$**

## 10 Surds and fractions

Simplify $\frac{1}{3 + \sqrt{7}} + \frac{1}{3 - \sqrt{7}}$
Give your answer with a rational denominator.

Common denominator: $(3 + \sqrt{7})(3 - \sqrt{7}) = 9 - 7 = 2$
Numerator: $(3 - \sqrt{7}) + (3 + \sqrt{7})$
$= 6$
$\frac{6}{2} = 3$

**Answer: 3**