Day 266 **S Tier 10-a-day: Model Solutions**

1 Weighted percentages

The ratio of boys to girls in a school is 3 : 2.
85% of the boys and 60% of the girls walk to school.
What percentage of all of these students walk to school?

Imagine 100 students in total: 60 boys and 40 girls
85% of 60 = 51 and 60% of 40 = 24
Together: 75 out of 100
75 ÷ 100 × 100 = 75%
Answer: 75%

2 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 50°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 50°
Angle COB is an exterior angle of triangle OAC: 50 + 50 = 100°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 100°

3 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 31 is taken. Work out how many come from Group B.
(b) A different stratified sample has 10 from Group B. Work out the size of this sample.

Total = 100 + 50 + 90 + 70 = 310

(a) Group B: $\frac{50}{310} \times 31 = 5$

(b) Fraction sampled = 10 ÷ 50 = $\frac{1}{5}$, so sample size = $\frac{1}{5} \times 310 = 62$

Answer: (a) 5 (b) 62

4 Transformations of graphs

The point P(4, 4) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of

(a) y = 2f(x) (b) y = $\frac{1}{2}$f(x)

(a) y = 2f(x) is a vertical stretch with scale factor 2, so the y coordinate is doubled: (4, 8)

(b) y = $\frac{1}{2}$f(x) is a vertical stretch with scale factor $\frac{1}{2}$, so the y coordinate is halved: (4, 2)

Answer: (a) (4, 8) (b) (4, 2)

5 Area of a triangle using sine

Triangle ABC has AB = 12 cm and BC = 20 cm.
The area of triangle ABC is 60 cm^2.
Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$60 = \frac{1}{2} \times 12 \times 20 \times \sin B$

$\sin B = 60 \div 120 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is acute, so B = 30°

Answer: 30°

Day 266 **S Tier 10-a-day: Model Solutions**

6 Multiplying and dividing surds

Simplify fully $2\sqrt{7} \times 3\sqrt{21}$

$2\sqrt{7} \times 3\sqrt{21} = 6\sqrt{147}$

$\sqrt{147} = 7\sqrt{3}$, so $6 \times 7\sqrt{3} = 42\sqrt{3}$

Answer: $42\sqrt{3}$

7 Tangent to a circle

A circle has equation $x^2 + y^2 = 100$.
The point P(8, 6) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (0, 0). The gradient of the radius to P is $\frac{6-0}{8-0} = \frac{3}{4}$

The tangent is perpendicular to the radius, so its gradient is $-\frac{4}{3}$. Using P: $y - 6 = -\frac{4}{3}(x - 8)$

Multiply by 3: $3(y - 6) = -4(x - 8)$
Expand and rearrange: $4x + 3y = 50$
Answer: $4x + 3y = 50$

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + 8x + 12}{x + 7} \div \frac{x + 2}{x^2 + 3x - 28}$

Factorise: $\frac{(x + 6)(x + 2)}{(x + 7)} \div \frac{(x + 2)}{(x + 7)(x - 4)}$

Turn the second fraction upside down and multiply: $\frac{(x + 6)(x + 2)}{(x + 7)} \times \frac{(x + 7)(x - 4)}{(x + 2)}$

Cancel (x + 2) and (x + 7)
$= (x - 4)(x + 6)$
Answer: $(x - 4)(x + 6)$

9 Probability from a frequency tree

In a population of 20,000 people, 4% have a certain disease.
A test gives a positive result for 80% of people who have the disease and for 5% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

4% of 20,000 = 800 have the disease and 19,200 do not
Flagged and genuine: 800 × 80% = 640
Flagged but not: 19,200 × 5% = 960
Total flagged or positive = 640 + 960 = 1,600

Probability = 640 ÷ 1,600 = $\frac{2}{5}$

Answer: $\frac{2}{5}$

10 Exact lengths and areas

A rectangle has area $(7 + 4\sqrt{3})$ cm^2 and length $(2 + \sqrt{3})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{3}$

Width = area ÷ length = $\frac{7 + 4\sqrt{3}}{2 + \sqrt{3}}$

Multiply top and bottom by $(2 - \sqrt{3})$:
Numerator: $(7 + 4\sqrt{3})(2 - \sqrt{3}) = 2 + \sqrt{3}$
Denominator: $4 - 3 = 1$
Width = $2 + \sqrt{3}$
Answer: $2 + \sqrt{3}$ cm