Day 265 **S Tier 10-a-day: Model Solutions**

1 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 50°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle OAB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 50 = 100°
(b) OA = OB (radii), so triangle OAB is isosceles
Angle OAB = (180 − 100) ÷ 2 = 40°
Answer: (a) 100° (b) 40°

2 Independent events and probability rules

A and B are two events.
P(A) = 0.8, P(B) = 0.4 and P(A or B) = 0.88
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.88 = 0.8 + 0.4 - P(A \cap B)$, so $P(A \cap B) = 0.32$
(b) $P(A) \times P(B) = 0.8 \times 0.4 = 0.32$
This equals $P(A \cap B)$, so A and B are independent
Answer: (a) 0.32 (b) Yes

3 Factorising fully

Factorise fully $27x^2 - 48y^2$

Take out the common factor 3: $3(9x^2 - 16y^2)$
$9x^2 - 16y^2$ is a difference of two squares: $(3x)^2 - (4y)^2$
$= (3x - 4y)(3x + 4y)$
Answer: $3(3x - 4y)(3x + 4y)$

4 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = (3x + 5)°, angle ACB = (2x + 20)° and angle CAQ = 51°.
(a) Work out the value of x.
(b) Work out the size of angle BAC.
The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)
2x + 20 = 3x + 5
x = 15
(b) Angle PAB = 50°
Angles on the straight line PAQ: angle BAC = 180 − 50 − 51 = 79°
Answer: (a) x = 15 (b) 79°

5 Exchange rates

Oscar changes £1200 into euros at £1 = €1.2.
Oscar spends €510 and then changes the rest back into pounds at £1 = €1.5.
How many pounds does Oscar get back?

£1200 × 1.2 = €1440
1440 − 510 = €930
930 ÷ 1.5 = £620
Answer: £620

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6 Where a line meets the axes

The line L has equation $12x + 5y = -60$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the length of AB.

(a) At A, $y = 0$: $12x = -60$, so $x = -5$. A is $(-5, 0)$
At B, $x = 0$: $5y = -60$, so $y = -12$. B is $(0, -12)$
(b) $AB^2 = 5^2 + 12^2 = 169$
$AB = \sqrt{169} = 13$
Answer: (a) A(−5, 0), B(0, −12) (b) 13

7 Speed-time graphs

The graph shows the speed of a train during 12 seconds.
(a) Work out the acceleration during the first 3 seconds.
(b) Work out the average speed over the whole 12 seconds.

(a) Acceleration = change in speed ÷ time = $(24 - 0) \div 3 = 8$ m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{0 + 24}{2} \times 3 = 36$; rectangle: $24 \times 4 = 96$; triangle: $\frac{1}{2} \times 5 \times 24 = 60$
Total = 192 m, so average speed = $192 \div 12 = 16$ m/s
Answer: (a) 8 m/s^2 (b) 16 m/s

8 HCF and LCM

The HCF of two numbers is 4 and their LCM is 308.
The smaller number is 28.
Work out the larger number.

HCF × LCM = product of the two numbers
$4 \times 308 = 1232$
Larger number = $1232 \div 28 = 44$
Answer: 44

9 Bearings with the sine and cosine rules

A plane leaves an airport P and travels 112 km on a bearing of 200° to Q. (Calculator)
It then travels 55 km on a bearing of 235° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is $200° - 180° = 020°$
Angle PQR = 145° (the difference between the bearings 020° and 235°)
$PR^2 = 112^2 + 55^2 - 2 \times 112 \times 55 \times \cos 145° = 25661$, so PR = 160.2 km
(b) $\cos(QPR) = \frac{112^2 + PR^2 - 55^2}{2 \times 112 \times PR}$, so angle QPR = 11.4°
R is clockwise of the direction PQ, so the bearing is 200° + 11° = 211°
Answer: (a) 160.2 km (b) 211°

10 Nested surds

Show that $\sqrt{70 + 24\sqrt{6}} = 4 + 3\sqrt{6}$

Square the right-hand side: $(4 + 3\sqrt{6})^2 = 16 + 24\sqrt{6} + 54$
$= 70 + 24\sqrt{6}$
So $\sqrt{70 + 24\sqrt{6}} = 4 + 3\sqrt{6}$
Answer: $\sqrt{70 + 24\sqrt{6}} = 4 + 3\sqrt{6}$