Day 264 **S Tier 10-a-day: Model Solutions**

1 Compound interest

Evan puts £20,000 in a savings account.
Compound interest is paid at 3% in year 1, 1% in year 2 and 3% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.03, 1.01, 1.03
£20,000 × 1.03 × 1.01 × 1.03
= £21,430.18
Answer: £21,430.18

2 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 21 is taken. Work out how many come from Year 8.
(b) A different stratified sample has 14 from Year 10. Work out the size of this sample.

Total = 40 + 30 + 70 + 70 = 210

(a) Year 8: $\frac{30}{210} \times 21 = 3$

(b) Fraction sampled = 14 ÷ 70 = $\frac{1}{5}$, so sample size = $\frac{1}{5} \times 210 = 42$

Answer: (a) 3 (b) 42

3 Arc length and sector area

A sector of a circle has angle 135° and area 24π cm^2.
(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{135}{360} \times \pi \times r^2$, so $\frac{135}{360} \times r^2 = 24$

$r^2 = 24 \times \frac{360}{135} = 64$

r = 8 cm

(b) Arc length = $\frac{135}{360} \times 2 \times \pi \times 8 = 6\pi$ cm

Answer: (a) 8 cm (b) 6π cm

4 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 4 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 16 ÷ 4 = 4 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 4 \times 16 + 16 \times 8 + \frac{1}{2} \times 8 \times 16 = 32 + 128 + 64$
= 224
Answer: (a) 4 m/s^2 (b) 224 m

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5 Column vectors

$\mathbf{a} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{c}$.

Work out the value of k.

$\mathbf{a} + k\mathbf{b} = \begin{pmatrix} -3 - 4k \\ 1 + 0k \end{pmatrix}$

Parallel to $\mathbf{c}$: $\frac{-3 - 4k}{4} = \frac{1 + 0k}{1}$

$1(-3 - 4k) = 4(1 + 0k)$

$k \times (-4) = 7$

$k = -\frac{7}{4}$

Answer: $\mathbf{k = -\frac{7}{4}}$

6 Quadratic graphs

The curve $y = x^2 + 6x - 16$ crosses the x-axis at the points A and B.

(a) Find the coordinates of A and B.

(b) Find the coordinates of the minimum point of the curve.

(a) Put $y = 0$: $x^2 + 6x - 16 = 0$, so $(x + 8)(x - 2) = 0$

$x + 8 = 0$ or $x - 2 = 0$, so $x = -8$ or $x = 2$

(b) The turning point is halfway between A and B: $x = (-8 + 2) \div 2 = -3$

When $x = -3$: $y = (-3)^2 + 6 \times (-3) - 16 = -25$

Answer: (a) A(−8, 0), B(2, 0) (b) (−3, −25)

7 Bounds of speed, density and pressure

A train travels a distance of 110 km, correct to the nearest 5.

The journey takes 9.6 hours, correct to 2 significant figures.

Work out the upper bound of the speed.

Give your answer correct to 3 significant figures. (Calculator)

$107.5 \leq$ distance < 112.5; $9.55 \leq$ time < 9.65

Upper bound of speed = biggest distance ÷ smallest time = 112.5 ÷ 9.55

= 11.7801... = 11.8 (3 s.f.)

Answer: 11.8 km/h

8 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 9 cm, DB = 6 cm and the area of triangle ADE is 72 cm^2.

Work out the area of the trapezium DBCE.

The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 9 + 6 = 15 cm

Length scale factor = $15 \div 9 = \frac{5}{3}$

Area scale factor = $(\frac{5}{3})^2 = \frac{25}{9}$

Area of triangle ABC = $72 \times \frac{25}{9} = 200$ cm^2

Trapezium = $200 - 72 = 128$ cm^2

Answer: 128 cm^2

9 Expanding brackets with surds

Show that $(5 + 2\sqrt{7})(5 - 2\sqrt{7})$ is an integer.

Use the difference of two squares: $(5 + 2\sqrt{7})(5 - 2\sqrt{7}) = 25 - (2\sqrt{7})^2$

$= 25 - 28 = -3$

Answer: −3

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10 Probability and quadratic equations

A bag contains some beads. 3 of them are blue and the rest are black.
Two beads are taken at random without replacement.

The probability that both are blue is $\frac{1}{5}$.

Work out how many beads are black.

Let there be n beads altogether

P(both blue) = $\frac{3}{n} \times \frac{2}{n-1} = \frac{6}{n(n-1)}$

$\frac{6}{n(n-1)} = \frac{1}{5}$, so $n(n-1) = 30$, giving $n^2 - n - 30 = 0$

$(n-6)(n+5) = 0$, so $n = 6$ (n cannot be negative)

Number of beads that are black = $6 - 3 = 3$

Answer: 3