Day 263 **S Tier 10-a-day: Model Solutions**

1 Area of a segment

The circle has centre O and radius 4 cm. A and B are points on the circle and angle AOB = 50°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{50}{360} \times \pi \times 4^2 = 6.9813$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 4 \times 4 \times \sin 50° = 6.1284$ cm^2

Segment = sector − triangle = 6.9813 − 6.1284 = 0.85296

= 0.853 cm^2 (3 s.f.)

Answer: 0.853 cm^2

2 Laws of indices with algebra

Simplify fully $\frac{2xy^5 \times 4xy}{2x^4y^5}$

Give your answer in the form kx^ay^b

Numbers: 2 × 4 ÷ 2 = 4

Powers of x: 1 + 1 − 4 = −2

Powers of y: 5 + 1 − 5 = 1

= $4x^{-2}y$

Answer: $4x^{-2}y$

3 Multiplying and dividing surds

Simplify fully $4\sqrt{45} \times 4\sqrt{7}$

$4\sqrt{45} \times 4\sqrt{7} = 16\sqrt{315}$

$\sqrt{315} = 3\sqrt{35}$, so $16 \times 3\sqrt{35} = 48\sqrt{35}$

Answer: $48\sqrt{35}$

4 Weighted percentages

The ratio of men to women in a school is 3 : 1.
65% of the men and 15% of the women own a car.
What percentage of all of these students own a car?

Imagine 80 students in total: 60 men and 20 women

65% of 60 = 39 and 15% of 20 = 3

Together: 42 out of 80

42 ÷ 80 × 100 = 52.5%

Answer: 52.5%

5 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 67°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 67 = 134°

(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)

Angle APB = 360 − 90 − 90 − 134 = 46°

Answer: (a) 134° (b) 46°

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6 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 50 − 30 = 20
IQR of B = 65 − 25 = 40
(b) Class B has the higher median (55 compared with 35), so on average Class B scored higher
Class A has the smaller interquartile range (20 compared with 40), so its marks are more consistent
Answer: (a) A: 20, B: 40 (b) B has the higher median, A has the smaller IQR

7 Terms of a quadratic sequence

The nth term of a sequence is $n^2 - n - 8$.
(a) Work out the 11th term.
(b) Is 201 a term of the sequence? You must show your working.

(a) $11^2 - 1 \times 11 - 8 = 102$
(b) Solve $n^2 - n - 8 = 201$, so $n^2 - n - 209 = 0$
$b^2 - 4ac = (-1)^2 - 4 \times 1 \times (-209) = 837$, which is not a square number
So n is not a whole number, and 201 is not a term
Answer: (a) 102 (b) No

8 Surds and fractions

Simplify $\frac{1}{3+\sqrt{7}} - \frac{1}{3-\sqrt{7}}$
Give your answer with a rational denominator.

Common denominator: $(3+\sqrt{7})(3-\sqrt{7}) = 9 - 7 = 2$
Numerator: $(3-\sqrt{7}) - (3+\sqrt{7})$
$= -2\sqrt{7}$
$\frac{-2\sqrt{7}}{2} = -\sqrt{7}$
Answer: $-\sqrt{7}$

9 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 3x - 18}{x - 2} \div \frac{x + 3}{x^2 - 3x + 2}$

Factorise: $\frac{(x-6)(x+3)}{(x-2)} \div \frac{(x+3)}{(x-2)(x-1)}$

Turn the second fraction upside down and multiply: $\frac{(x-6)(x+3)}{(x-2)} \times \frac{(x-2)(x-1)}{(x+3)}$

Cancel (x + 3) and (x − 2)
$= (x-6)(x-1)$
Answer: (x − 6)(x − 1)

10 Bounds of roots

The area of a square is 333 cm^2, correct to the nearest cm^2.
Work out the lower bound of the side length of the square.
Give your answer correct to 3 significant figures. (Calculator)

$332.5 \leq$ area < 333.5
Lower bound of side $= \sqrt{332.5}$
$= 18.2346... = 18.2$ (3 s.f.)
Answer: 18.2 cm