Day 262 **S Tier 10-a-day: Model Solutions**

1 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(4x - 10)°$
Angle PQR = $(2x + 10)°$
Angle PRS = $(5x + 17)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$5x + 17 = 6x$
$x = 17$
(b) Angle PRS = 102° so angle PRQ = $180° - 102° = 78°$
Answer: (a) x = 17 (b) 78°

2 Fractions of fractions

Ravi cycles $\frac{8}{9}$ of a route before lunch.

After lunch he cycles $\frac{1}{3}$ of the remaining distance.

He still has 14 km to go.
How long is the route, in km?

After the first part, $\frac{1}{9}$ remains

After the second, $\frac{2}{3}$ of $\frac{1}{9}$ remains = $\frac{2}{27}$ of the total

$\frac{2}{27}$ of the total is 14

Total = $14 \div \frac{2}{27} = 189$

Answer: 189 km

3 Enlargement

Triangle T has vertices A(−4, 0), B(−2, −4), C(8, 4).

T is enlarged with scale factor $\frac{3}{2}$ and centre (2, −2).

(a) Write down the coordinates of the image of A.
(b) The area of T is 18 square units. Work out the area of the image of T.

(a) Vector from the centre to A = $(-4 - 2, 0 - (-2)) = (-6, 2)$

Multiply by $\frac{3}{2}$: $(-9, 3)$

Image of A = centre + this vector = $(-7, 1)$

(b) Area scale factor = $(\frac{3}{2})^2 = \frac{9}{4}$

Area = $18 \times \frac{9}{4} = \frac{81}{2}$ square units

Answer: (a) (−7, 1) (b) $\frac{81}{2}$

4 Completing the square

(a) Write $x^2 + 12x + 31$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 + 12x + 31$

(a) Half of 12 is 6, so $x^2 + 12x = (x + 6)^2 - 36$
$x^2 + 12x + 31 = (x + 6)^2 - 36 + 31$
$= (x + 6)^2 - 5$
(b) The minimum is when $x + 6 = 0$, so $x = -6$ and $y = -5$
Answer: (a) $(x + 6)^2 - 5$ (b) (−6, −5)

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5 Speed-time graphs

The graph shows the speed of a train during 20 seconds.
(a) Work out the acceleration during the first 6 seconds.
(b) Work out the average speed over the whole 20 seconds.

(a) Acceleration = change in speed ÷ time = $(24 - 6) \div 6 = 3$ m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{6 + 24}{2} \times 6 = 90$; rectangle: $24 \times 10 = 240$; triangle: $\frac{1}{2} \times 4 \times 24 = 48$
Total = 378 m, so average speed = $378 \div 20 = 18.9$ m/s
Answer: (a) 3 m/s^2 (b) 18.9 m/s

6 Profit and loss

A retailer buys a watch for £340.
It adds a 20% mark-up to get the selling price.
In a sale the selling price is reduced by 25%.
Work out the percentage profit or loss on the original cost.

Marked price = £340 × 1.2 = £408
Sale price = £408 × 0.75 = £306
Loss = £34
Percentage = $\frac{34}{340} \times 100 = 10\%$
Answer: 10% loss

7 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 47° and angle CAQ = 32°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 32° (alternate segment theorem)
(b) Angle ACB = angle PAB = 47° (alternate segment theorem)
Answer: (a) 32° (b) 47°

8 Equation of a circle

A circle has centre C(−3, −4) and passes through the point P(−6, 5).
Find an equation of the circle.

$r^2 = CP^2 = (-6 - (-3))^2 + (5 - (-4))^2$
$= 9 + 81 = 90$
$(x + 3)^2 + (y + 4)^2 = 90$
Answer: $(x + 3)^2 + (y + 4)^2 = 90$

9 Moving averages

The table shows the daily visitors over six periods.
The 3-point moving average centred on Thu is 29.
(a) Work out the value of x.
(b) Work out the 3-point moving average centred on Fri.

(a) $(23 + x + 36) \div 3 = 29$
$23 + x + 36 = 87$, so $x = 28$
(b) $(28 + 36 + 23) \div 3 = 87 \div 3 = 29$
Answer: (a) x = 28 (b) 29

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10 Substitution

Work out the value of $\frac{2x - y}{z}$

when x = 1, y = 2 and z = 2.

Substitute the values: $\frac{2 \times 1 - 2}{2}$

$= \frac{0}{2}$

$= 0$

Answer: 0