Day 261 **S Tier 10-a-day: Model Solutions**

1 Area and quadratic equations

The diagram shows a rectangle of length $(x - 1)$ cm, width $(2x + 1)$ cm and area 65 cm^2.
(a) Show that $2x^2 - x - 66 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(x - 1)(2x + 1) = 65$
Expand: $2x^2 - x - 1 = 65$, so $2x^2 - x - 66 = 0$
(b) Factorise: $(x - 6)(2x + 11) = 0$
$x = 6$ ($x = -\frac{11}{2}$ is not possible as a length would be negative)
Length = 5 cm and width = 13 cm
Perimeter = $2 \times (5 + 13) = 36$ cm
Answer: (b) 36 cm

2 Tests of divisibility

The 5-digit number 5376k is a multiple of 8.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 8 if its last three digits form a multiple of 8
Last three digits: 76k
Multiples of 8 of this form: 760, 768
Answer: k = 0 or 8

3 Similar shapes

Two shapes are mathematically similar.
The smaller has height 24 cm and area 36 cm^2.
The larger has area 64 cm^2.
Work out the height of the larger shape.

Ratio of areas = 64 : 36 = 16 : 9
Ratio of lengths = 4 : 3
Height = $24 \times \frac{4}{3} = 32$ cm
Answer: 32 cm

4 Equations with indices

Solve $x^{\frac{4}{3}} = 16$
$x > 0$

Write 16 as a power of 2: $16 = 2^4$
$x^{\frac{4}{3}} = 2^4$, so $x = (2^4)^{\frac{3}{4}}$
$x = 2^3 = 8$
Answer: x = 8

5 Pressure

A wooden block in the shape of a cuboid weighs 90 N.
Its smallest face has area 100 cm^2. Its largest face has area 1,200 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the wooden block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 100 cm^2 = 0.01 m^2 and 1,200 cm^2 = 0.12 m^2
(a) Greatest pressure is on the smallest face: $90 \div 0.01 = 9{,}000$ N/m^2
(b) Least pressure is on the largest face: $90 \div 0.12 = 750$ N/m^2
Answer: (a) 9,000 N/m^2 (b) 750 N/m^2

Day 261 **S Tier 10-a-day: Model Solutions**

6 Simple and compound interest

Ivan invests £8,000 for 5 years at 1.5% per year simple interest.
Omar invests £8,000 for 5 years at 1.5% per year compound interest.
Work out how much more interest Omar earns than Ivan.
Give your answer to the nearest penny. (Calculator)

Simple interest: £8,000 × 1.5% × 5 = £600
Compound: £8,000 × 1.015^5 = £8,618.27
Compound interest = £8,618.27 − £8,000 = £618.27
Difference = £618.27 − £600 = £18.27
Answer: £18.27

7 Sample space and conditional probability

A fair 6-sided dice and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the product is greater than 8.
(b) Given that at least one number is 3, work out the probability that the product is greater than 8.

(a) There are 6 × 5 = 30 equally likely outcomes, and 15 of them satisfy: the product is greater than 8
Probability = $\frac{1}{2}$
(b) 10 outcomes satisfy: at least one number is 3. Of these, 6 also satisfy: the product is greater than 8
Probability = $\frac{3}{5}$
Answer: (a) $\frac{1}{2}$ (b) $\frac{3}{5}$

8 Iteration: rearranging equations

(a) Show that the equation $x^3 - 3x - 40 = 0$ can be rearranged to give $x = \sqrt[3]{3x + 40}$ (Calculator)
(b) Use the iteration formula $x_{n+1} = \sqrt[3]{3x_n + 40}$ with $x_0 = 3$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^3 - 3x - 40 = 0$: $x^3 = 3x + 40$
Take the cube root of both sides: $x = \sqrt[3]{3x + 40}$
(b) $x_1 = \sqrt[3]{3 \times 3 + 40} = 3.65931...$
$x_2 = \sqrt[3]{3 \times x_1 + 40} = 3.70789...$
$x_1 = 3.659$ and $x_2 = 3.708$ (3 d.p.)
Answer: (b) $x_1 = 3.659$, $x_2 = 3.708$

9 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 35.
Work out the value of x.

Mid-points: 15, 25, 35, 45, 55
Estimated mean = Σfm ÷ Σf, so (1500 + 25x) ÷ (42 + x) = 35
1500 + 25x = 35(42 + x) = 1470 + 35x
(25 − 35)x = 1470 − 1500
−10x = −30, so x = 3
Answer: x = 3

Day 261 **S Tier 10-a-day: Model Solutions**

10 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{2-0}{0-8} = -\frac{1}{4}$

Perpendicular gradient = $-1 \div (-\frac{1}{4}) = 4$

M passes through B(8, 0): $0 = 4 \times 8 + c$, so $c = -32$
$y = 4x - 32$
Answer: $y = 4x - 32$