Day 260 **S Tier 10-a-day: Model Solutions**

1 Comparing costs

Hire Shop A has a delivery charge of £5.50 plus £6 per day.
Hire Shop B has a delivery charge of £25.50 plus £4 per day.
(a) Work out the number of days for which the two costs are the same.
(b) Which is cheaper for 19 days, and by how much?

(a) Let n be the number of days. Working in pounds, the costs are equal when 5.50 + 6n = 25.50 + 4n
2n = 20, so n = 10
(b) Hire Shop A: £5.50 + £6 × 19 = £119.50
Hire Shop B: £25.50 + £4 × 19 = £101.50
Hire Shop B is cheaper by £18
Answer: (a) 10 (b) Hire Shop B, by £18

2 Capture and recapture

80 fish in a lake are caught, tagged and put back.
Later, a second sample of 60 fish is caught. 4 of these are tagged.
(a) Work out an estimate for the number of fish in a lake.
(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 4 out of 60

This should match the proportion tagged in the whole population: $\frac{4}{60} = \frac{80}{N}$

N = 80 × 60 ÷ 4 = 1200
(b) No fish are born, die or leave between the samples, and the fish mix evenly
Answer: (a) 1200 (b) The population stays the same between samples

3 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 34° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin\theta}{AC} = \frac{\sin A}{BC}$

$\frac{\sin\theta}{9} = \frac{\sin 34°}{16}$

$\sin\theta = \frac{9 \times \sin 34°}{16} = 0.31455$

$\theta = \sin^{-1}(0.31455) = 18.3°$ (1 d.p.)
Answer: θ = 18.3°

4 Fractional and negative indices

Work out $81^{\frac{7}{4}}$

The denominator 4 means the fourth root: the fourth root of 81 is 3
The numerator 7 means raise to the power 7: $(3)^7 = 2187$
Answer: 2187

5 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 68° and angle CAQ = 40°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 40° (alternate segment theorem)
(b) Angle ACB = angle PAB = 68° (alternate segment theorem)
Answer: (a) 40° (b) 68°

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6 Algebraic proof

Prove that $(k + 4)^2 - (k - 4)^2$ is a multiple of 16 for all integer values of k.

$(k + 4)^2 = k^2 + 8k + 16$ and $(k - 4)^2 = k^2 - 8k + 16$

Subtract: $(k + 4)^2 - (k - 4)^2 = 16k$

$16k = 16 \times k$, which is a multiple of 16 because k is an integer

Answer: Proved

7 Counting factors

How many factors does 288 have?
Include 1 and 288.

$288 = 2^5 \times 3^2$

Add 1 to each power and multiply: $6 \times 3 = 18$

Answer: 18

8 Ratio and money transferred

Callum and Uma share some money in the ratio 5 : 8.
Callum gives Uma £2.
The ratio of their money is now 3 : 5.
How much money do they have altogether?

Let the amounts be 5k and 8k, so the total is 13k

Callum now has $5k - 2$, which is $\frac{3}{8}$ of the total

$8(5k - 2) = 3 \times 13k$

$40k - 16 = 39k$, so $1k = 16$ and $k = 16$

Total $= 13 \times 16 = £208$

Answer: £208

9 Line and curve intersections

Solve the simultaneous equations

$y = x^2 + 3x + 5$
$y = -4x - 7$

Substitute: $x^2 + 3x + 5 = -4x - 7$

Rearrange: $x^2 + 7x + 12 = 0$

Factorise: $(x + 3)(x + 4) = 0$, so $x = -3$ or $x = -4$

When $x = -3$: $y = -4 \times (-3) - 7 = 5$

When $x = -4$: $y = -4 \times (-4) - 7 = 9$

Answer: $x = -4$, $y = 9$ and $x = -3$, $y = 5$

10 Bounds and accuracy

a = 0.389 and b = 5.74, both correct to 3 significant figures.
Work out a × b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$0.3885 \leq a < 0.3895$; $5.735 \leq b < 5.745$

Lower bound $= 0.3885 \times 5.735 = 2.2280475$

Upper bound $= 0.3895 \times 5.745 = 2.2376775$

Both bounds round to 2.2 to 2 significant figures, but not to 3 significant figures

Answer: 2.2 (2 s.f.)