Day 26 **S Tier 10-a-day: Model Solutions**

1 Relative speed

Hassan sets off at 8:00 and cycles at 24 km/h along a path.
Priya sets off from the same place at 9:00 and cycles at 34 km/h along the same path.
At what time does Priya catch up with Hassan?

When Priya starts, Hassan is 1 × 24 = 24 km ahead
Priya gains 34 − 24 = 10 km every hour
Time to catch up = 24 ÷ 10 = 2.4 hours = 144 minutes
9:00 + 144 minutes = 11:24
Answer: 11:24

2 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 61°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 61°
Angle COB is an exterior angle of triangle OAC: 61 + 61 = 122°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 122°

3 Simultaneous equations

Solve the simultaneous equations
$2x - y = 7$
$-3x - 5y = -4$

To match the y terms: (1) × 5 gives $10x - 5y = 35$
Subtract the equations to eliminate y: $13x = 39$
$x = 3$
Substitute x = 3 into (1): $-y + 6 = 7$
$-y = 1$, so $y = -1$
Answer: $x = 3$, $y = -1$

4 Pressure

A concrete slab in the shape of a cuboid weighs 960 N.
Its smallest face has area 200 cm^2. Its largest face has area 2,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 200 cm^2 = 0.02 m^2 and 2,000 cm^2 = 0.2 m^2
(a) Greatest pressure is on the smallest face: 960 ÷ 0.02 = 48,000 N/m^2
(b) Least pressure is on the largest face: 960 ÷ 0.2 = 4,800 N/m^2
Answer: (a) 48,000 N/m^2 (b) 4,800 N/m^2

5 Expanding brackets with surds

Show that $(6 - 3\sqrt{3})(6 + 3\sqrt{3})$ is an integer.

Use the difference of two squares: $(6 - 3\sqrt{3})(6 + 3\sqrt{3}) = 36 - (3\sqrt{3})^2$
$= 36 - 27 = 9$
Answer: 9

6 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 8 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 32 ÷ 8 = 4 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 8 \times 32 + 32 \times 12 + \frac{1}{2} \times 8 \times 32 = 128 + 384 + 128$
$= 640$
Answer: (a) 4 m/s^2 (b) 640 m

7 Radius from chord and height of arc

A circular arc has a chord AB of length 18 cm.
The greatest distance between the chord and the arc is 1 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 1
AM = 18 ÷ 2 = 9. Pythagoras in triangle OMA: $r^2 = (r - 1)^2 + 9^2$
$r^2 = r^2 - 2r + 1 + 81$
$2r = 82$, so r = 41 cm
Answer: 41 cm

8 Cone made from a sector

A sector of a circle has radius 10 cm and angle 288°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{288}{360} \times 2\pi \times 10 = 16\pi$ cm
This is the circumference of the base: $2\pi r = 16\pi$, so r = 8 cm
(b) Slant height = 10 cm, so height$^2 = 10^2 - 8^2 = 36$, height = 6 cm
Volume = $\frac{1}{3} \times \pi \times 8^2 \times 6 = 128\pi$ cm^3
Answer: (a) 8 cm (b) 128π cm^3

9 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are x = −2 and x = 6, so y = a(x + 2)(x − 6)
When x = 0, y = −24: a × 2 × (−6) = −24, so a = 2
y = 2(x + 2)(x − 6)
$y = 2x^2 - 8x - 24$
Answer: $y = 2x^2 - 8x - 24$

10 Nested surds

Show that $\sqrt{21 + 4\sqrt{5}} = 1 + 2\sqrt{5}$

Square the right-hand side: $(1 + 2\sqrt{5})^2 = 1 + 4\sqrt{5} + 20$
$= 21 + 4\sqrt{5}$
So $\sqrt{21 + 4\sqrt{5}} = 1 + 2\sqrt{5}$
Answer: $\sqrt{21 + 4\sqrt{5}} = 1 + 2\sqrt{5}$