Day 259 **S Tier 10-a-day: Model Solutions**

1 Tests of divisibility

The 4-digit number 4k01 is a multiple of 3.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 3 if its digit sum is a multiple of 3
Digit sum = 4 + 0 + 1 + k = 5 + k
This must be a multiple of 3, so k = 1 or 4 or 7
Answer: k = 1 or 4 or 7

2 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 10 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 30 ÷ 10 = 3 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 10 \times 30 + 30 \times 8 + \frac{1}{2} \times 30 \times 30 = 150 + 240 + 450$
= 840
Answer: (a) 3 m/s^2 (b) 840 m

3 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 6 cm and inner radius 5 cm. (Calculator)
The density of the metal is 11.3 g/cm^3.
The mass of the pipe is 31.2 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 31200 g, so volume = mass ÷ density = 31200 ÷ 11.3 = 2761.1 cm^3
Cross-sectional area = $\pi \times 6^2 - \pi \times 5^2$ = 34.558 cm^2
Length = volume ÷ area = 2761.1 ÷ 34.558 = 79.9 cm (3 s.f.)
Answer: 79.9 cm

4 Expanding brackets with surds

Expand and simplify $(6 - \sqrt{3})^2$
Give your answer in the form $p + q\sqrt{3}$

$(6 - \sqrt{3})^2 = 36 - 12\sqrt{3} + 3$
$= 39 - 12\sqrt{3}$
Answer: $39 - 12\sqrt{3}$

5 Equations with indices

Solve $5^{2x-1} = \frac{1}{625}$

Write the right-hand side as a power of 5: $\frac{1}{625} = 5^{-4}$
Equate the powers: $2x - 1 = -4$
$2x = -3$
$x = -\frac{3}{2}$
Answer: $x = -\frac{3}{2}$

Day 259 **S Tier 10-a-day: Model Solutions**

6 Conditional probability from a table

The table shows information about 73 students.
(a) A student who studies German is chosen at random. Work out the probability that this student is a boy.
(b) Two of the 73 students are chosen at random. Work out the probability that both are boys.

(a) 11 students are in the German column, and 5 of them are boys

Probability = $\frac{5}{11}$

(b) There are 34 boys, so P(both) = $\frac{34}{73} \times \frac{33}{72} = \frac{187}{876}$

Answer: (a) $\frac{5}{11}$ (b) $\frac{187}{876}$

7 Pythagoras with algebra

ABC is a right-angled triangle with sides (x + 4) cm, (x − 3) cm and (x + 5) cm. The longest side is (x + 5) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 4)^2 + (x - 3)^2 = (x + 5)^2$
Expand and collect terms: $x^2 - 8x = 0$
Factorise: $x(x - 8) = 0$
x = 8 or x = 0, and x must be positive, so x = 8
(b) The two shorter sides are 12 cm and 5 cm

Area = $\frac{1}{2} \times 12 \times 5 = 30$ cm^2

Answer: (a) x = 8 (b) 30 cm^2

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 169$.
The point P(−5, −12) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{12}{5}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{12}{5} = -\frac{5}{12}$

$y - (-12) = (-\frac{5}{12})(x - (-5))$

Multiply out and rearrange: $5x + 12y = -169$
(b) On the x-axis y = 0, so $5x = -169$

$x = -\frac{169}{5}$, so T is $(-\frac{169}{5}, 0)$

Answer: (a) $5x + 12y = -169$ (b) T = $(-\frac{169}{5}, 0)$

Day 259 **S Tier 10-a-day: Model Solutions**

9 Perpendicular bisector

A is the point (8, −7) and B is the point (14, −17).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form y = mx + c.

Midpoint of AB = $(\frac{8+14}{2}, \frac{-7+(-17)}{2}) = (11, -12)$

Gradient of AB = $\frac{-17-(-7)}{14-8} = -\frac{5}{3}$

The perpendicular gradient is $-1 \div (-\frac{5}{3}) = \frac{3}{5}$

Using the midpoint: $y + 12 = \frac{3}{5}(x - 11)$

$y = \frac{3}{5}x - \frac{93}{5}$

Answer: $y = \frac{3}{5}x - \frac{93}{5}$

10 Reverse percentage growth and decay

A car loses 20% of its value each year.
After 3 years it is worth £3,584.
Work out its value when it was new. (Calculator)

Yearly multiplier = 0.8

Original × 0.8^3 = £3,584

Original = £3,584 ÷ 0.8^3 = £7,000

Answer: £7,000