Day 258 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The sum of the interior angles of a polygon is 3420°.
Work out the number of sides of the polygon.

Sum of interior angles = $(n - 2) \times 180°$
$(n - 2) \times 180 = 3420$
$n - 2 = 3420 \div 180 = 19$
$n = 21$
Answer: 21

2 Factorising quadratics

Factorise $12x^2 + 19x + 5$

$12 \times 5 = 60$. Two numbers that multiply to 60 and add to 19 are 4 and 15
Split the middle term: $12x^2 + 4x + 15x + 5$
Factorise in pairs: $4x(3x + 1) + 5(3x + 1)$
$= (4x + 5)(3x + 1)$
Answer: (4x + 5)(3x + 1)

3 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 48° and angle CAQ = 52°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 52° (alternate segment theorem)
(b) Angle ACB = angle PAB = 48° (alternate segment theorem)
Answer: (a) 52° (b) 48°

4 Trigonometric graphs

The diagram shows the graph of $y = a \cos x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At $x = 0$, $\cos x° = 1$, so $a + c = 6$
At $x = 180$, $\cos x° = -1$, so $-a + c = 4$
Add: $2c = 10$, so $c = 5$
Subtract: $2a = 2$, so $a = 1$
Answer: a = 1, c = 5

5 Ratio and algebra

In a fruit box, the ratio of the number of apples to the number of pears is 5 : 2.
Then 19 apples are removed and 20 more pears are added.
The ratio is now 6 : 7.
How many apples were there at the start?

Let the numbers at the start be 5k and 2k
$(5k - 19) : (2k + 20) = 6 : 7$, so $7(5k - 19) = 6(2k + 20)$
$35k - 133 = 12k + 120$
$23k = 253$, so $k = 11$
Number of apples at the start: $5 \times 11 = 55$
Answer: 55

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6 Standard form

Work out $(9.2 \times 10^5) \times (5.2 \times 10^3)$
Give your answer in standard form.

$9.2 \times 5.2 = 47.84$

$10^5 \times 10^3 = 10^8$

$47.84 \times 10^8 = 4.784 \times 10^9$

Answer: 4.784×10^9

7 Relative speed

Noah sets off at 9:00 and cycles at 24 km/h along a path.
Sofia sets off from the same place at 9:15 and cycles at 30 km/h along the same path.
At what time does Sofia catch up with Noah?

When Sofia starts, Noah is $0.25 \times 24 = 6$ km ahead
Sofia gains $30 - 24 = 6$ km every hour
Time to catch up $= 6 \div 6 = 1$ hours $= 60$ minutes
9:15 + 60 minutes = 10:15
Answer: 10:15

8 Exact trigonometric values

Work out the exact value of $\cos 45° \div \tan 30°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\cos 45° \div \tan 30° = \frac{\sqrt{2}}{2} \div \frac{\sqrt{3}}{3}$

$= \frac{\sqrt{6}}{2}$

Answer: $\frac{\sqrt{6}}{2}$

9 Rearranging when the subject appears twice

Make x the subject of $y(x + 6) = x - y$

Expand the bracket: $yx + 6y = x - y$
Collect the x terms: $yx - x = -6y - y$
Factorise: $x(y - 1) = -7y$

$x = \frac{7y}{1 - y}$

Answer: $x = \frac{7y}{1 - y}$

10 Surds with unknowns

Given that $(a + \sqrt{11})^2 = 12 + 2\sqrt{11}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{11})^2 = a^2 + 2a\sqrt{11} + 11$

Compare the $\sqrt{11}$ terms: $2a = 2$, so $a = 1$

Check the integer terms: $a^2 + 11 = 1 + 11 = 12$

Answer: $a = 1$