Day 257 **S Tier 10-a-day: Model Solutions**

1 Tests of divisibility

The 4-digit number 737k is a multiple of 8.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 8 if its last three digits form a multiple of 8
Last three digits: 37k
Multiples of 8 of this form: 376
Answer: k = 6

2 Mean of a changing set

The mean of 8 numbers is 98.
One number is removed and the mean of the remaining 7 numbers is 106.
Work out the number that was removed.

Total of the 8 numbers = 8 × 98 = 784
Total of the 7 remaining = 7 × 106 = 742
Number removed = 784 − 742 = 42
Answer: 42

3 Column vectors

$\mathbf{a} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$
$\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{c}$.
Work out the value of k.

$\mathbf{a} + k\mathbf{b} = \begin{pmatrix} 0 + 0k \\ -4 + 2k \end{pmatrix}$
Parallel to $\mathbf{c}$: $\frac{0 + 0k}{3} = \frac{-4 + 2k}{-2}$
$-2(0 + 0k) = 3(-4 + 2k)$
$k \times (-6) = -12$
$k = 2$
Answer: k = 2

4 Percentages of percentages

In a group of members, 35% are juniors and the rest are seniors.
15% of the juniors and 30% of the seniors play chess.
What percentage of all the members play chess?

Take 100 members: 35 are juniors and 65 are seniors
15% of 35 = 5.25 and 30% of 65 = 19.5
5.25 + 19.5 = 24.75 out of 100
Answer: 24.75%

5 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 50 cm. (Calculator)
The outer radius is 4 cm and the inner radius is 2 cm.
The density of the metal is 11.3 g/cm^3.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 4^2 \times 50 - \pi \times 2^2 \times 50 = \pi(16 - 4) \times 50$
$= 1885.0$ cm^3
Mass = density × volume = 11.3 × 1885.0 = 21300 g
= 21.3 kg (3 s.f.)
Answer: 21.3 kg

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6 Ratio and algebra

In a club, the ratio of the number of boys to the number of girls is 2 : 7.
Then 3 more boys are added and 16 girls are removed.
The ratio is now 11 : 12.
How many boys were there at the start?

Let the numbers at the start be 2k and 7k
(2k + 3) : (7k − 16) = 11 : 12, so 12(2k + 3) = 11(7k − 16)
24k + 36 = 77k − 176
−53k = −212, so k = 4
Number of boys at the start: 2 × 4 = 8
Answer: 8

7 Composite functions: fg and gf

f(x) = 3x + 1 and g(x) = x^2
Solve fg(x) = gf(x)

fg(x) = f(x^2) = $3x^2 + 1$
gf(x) = g(3x + 1) = $(3x + 1)^2 = 9x^2 + 6x + 1$
Set them equal and rearrange: $6x^2 + 6x = 0$
Divide by 6 and factorise: x(x + 1) = 0
x = −1 or x = 0
Answer: x = −1 or x = 0

8 Direct and inverse proportion

T is directly proportional to the cube of l.
When l = 5, T = 1000.
(a) Find a formula for T in terms of l.
(b) Find the value of T when l = 6.

(a) $T = kl^3$
Substitute l = 5 and T = 1000: $1000 = k \times 5^3$
k = 8
$T = 8l^3$
(b) $T = 86^3 = 1728$
Answer: (a) $T = 8l^3$ (b) 1728

9 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the opposite side BC and the known side is the hypotenuse AC, so use sin 30°
$\sin 30° = \frac{1}{2}$
$x = 4\sqrt{3} \times \sin 30° = 4\sqrt{3} \times \frac{1}{2}$
$x = 2\sqrt{3}$
Answer: $x = 2\sqrt{3}$

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10 Probability from a frequency tree

In a population of 10,000 people, 4% have a certain disease.
A test gives a positive result for 80% of people who have the disease and for 2% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

4% of 10,000 = 400 have the disease and 9,600 do not
Flagged and genuine: 400 × 80% = 320
Flagged but not: 9,600 × 2% = 192
Total flagged or positive = 320 + 192 = 512

Probability = 320 ÷ 512 = $\frac{5}{8}$

Answer: $\mathbf{\frac{5}{8}}$