Day 256 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(2x)°$, $(x + 10)°$, $(x - 15)°$, $(x + 35)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$6x + 30 = 540$, so $6x = 510$
$x = 85$
(b) The angles are 85°, 170°, 95°, 70°, 120°
The largest is 170°
Answer: (a) x = 85 (b) 170°

2 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 32 is taken. Work out how many come from Group C.
(b) A different stratified sample has 33 from Group B. Work out the size of this sample.

Total = 180 + 220 + 100 + 140 = 640

(a) Group C: $\frac{100}{640} \times 32 = 5$

(b) Fraction sampled = $33 \div 220 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 640 = 96$

Answer: (a) 5 (b) 96

3 Iteration

Use the iteration formula $x_{n+1} = \sqrt[3]{3x_n + 2}$ with $x_0 = 4$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt[3]{3 \times 4 + 2} = 2.41014...$
$x_2 = \sqrt[3]{3 \times x_1 + 2} = 2.09769...$
$x_3 = \sqrt[3]{3 \times x_2 + 2} = 2.02413...$
Rounded: $x_1 = 2.410$, $x_2 = 2.098$, $x_3 = 2.024$
Answer: $x_1 = 2.410$, $x_2 = 2.098$, $x_3 = 2.024$

4 Combined transformations

The point P(−5, −6) is reflected in the x-axis.
The image is then reflected in the line y = x.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−5, 6)
After the second transformation: P'' = (6, −5)
(b) Follow a general point (x, y) through both transformations: (x, y) → (x, −y) → (−y, x)
The single mapping is (x, y) → (−y, x)
This is a rotation 90° anticlockwise about the origin
Answer: (a) (6, −5) (b) Rotation 90° anticlockwise about the origin

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5 Cumulative frequency graphs

The cumulative frequency graph gives information about the marks of 120 students in a test.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 120 had a mark greater than 45.

(a) Median at cumulative frequency 60, so median = 30
Lower quartile at 30, so LQ = 20. Upper quartile at 90, so UQ = 40
Interquartile range = 40 − 20 = 20
(b) At 45, cumulative frequency = 94
Number greater = 120 − 94 = 26
Answer: (a) median 30, IQR 20 (b) 26

6 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the hypotenuse AC, so use cos 30°

$\cos 30° = \frac{\sqrt{3}}{2}$

$x = 7\sqrt{3} \times \cos 30° = 7\sqrt{3} \times \frac{\sqrt{3}}{2}$

$x = \frac{21}{2}$

Answer: $\mathbf{x = \frac{21}{2}}$

7 Line and curve intersections

Solve the simultaneous equations
$y = x^2 - x - 5$
$y = -x + 4$

Substitute: $x^2 - x - 5 = -x + 4$
Rearrange: $x^2 - 9 = 0$
Factorise: $(x + 3)(x - 3) = 0$, so $x = -3$ or $x = 3$
When $x = -3$: $y = -1 \times (-3) + 4 = 7$
When $x = 3$: $y = -1 \times 3 + 4 = 1$
Answer: x = −3, y = 7 and x = 3, y = 1

8 Growth and decay

Omar puts £1,000 into a savings account.
The account pays 6% compound interest each year.
After how many whole years will there first be more than £1,350 in the account? (Calculator)

Multiplier each year = 1.06
After 5 years: £1,000 × 1.06^5 = £1,338.23
After 6 years: £1,000 × 1.06^6 = £1,418.52
So the first time it is more than £1,350 is after 6 years
Answer: 6 years

9 Rearranging when the subject appears twice

Make t the subject of $T(t + 4) = t - T$

Expand the bracket: $Tt + 4T = t - T$
Collect the t terms: $Tt - t = -4T - T$
Factorise: $t(T - 1) = -5T$
$t = \frac{5T}{1 - T}$

Answer: $\mathbf{t = \frac{5T}{1 - T}}$

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10 Exact lengths and areas

A rectangle has area $(6 + \sqrt{2})$ cm^2 and length $(2 + \sqrt{2})$ cm.
Find the width of the rectangle.

Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\dfrac{6 + \sqrt{2}}{2 + \sqrt{2}}$

Multiply top and bottom by $(2 - \sqrt{2})$:

Numerator: $(6 + \sqrt{2})(2 - \sqrt{2}) = 10 - 4\sqrt{2}$

Denominator: $4 - 2 = 2$

Width = $5 - 2\sqrt{2}$

Answer: $5 - 2\sqrt{2}$ cm