Day 255 **S Tier 10-a-day: Model Solutions**

1 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (2x + 20)°
Angle ABC = (x + 34)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(2x + 20) + 2(x + 34) = 180
4x + 88 = 180, so 4x = 92
x = 23
(b) Angle BAC = 2 × 23 + 20 = 66°
Answer: (a) x = 23 (b) 66°

2 Simplifying surds

Write $\sqrt{48} + \sqrt{3} - \sqrt{12}$ in the form $a\sqrt{3}$
where a is an integer.

$\sqrt{48} = 4\sqrt{3}$; $\sqrt{12} = 2\sqrt{3}$
$(4 + 1 - 2)\sqrt{3} = 3\sqrt{3}$
Answer: $3\sqrt{3}$

3 Completing the square

(a) Write $x^2 - 2x + 9$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 - 2x + 9$

(a) Half of −2 is −1, so $x^2 - 2x = (x - 1)^2 - 1$
$x^2 - 2x + 9 = (x - 1)^2 - 1 + 9$
$= (x - 1)^2 + 8$
(b) The minimum is when $x - 1 = 0$, so x = 1 and y = 8
Answer: (a) $(x - 1)^2 + 8$ (b) (1, 8)

4 Fractions

Work out $3\frac{2}{5} \times 2\frac{1}{2} - 4\frac{3}{4}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions
$\frac{17}{5} \times \frac{5}{2} = \frac{17}{2}$
$\frac{17}{2} - \frac{19}{4} = \frac{15}{4}$
$= 3\frac{3}{4}$
Answer: $3\frac{3}{4}$

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5 Area of a triangle using sine

Triangle ABC has AB = 7 cm and BC = 14 cm.

The area of triangle ABC is $\frac{49\sqrt{2}}{2}$ cm^2.

Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$\frac{49\sqrt{2}}{2} = \frac{1}{2} \times 7 \times 14 \times \sin B$

$\sin B = \frac{49\sqrt{2}}{2} \div 49 = \frac{\sqrt{2}}{2}$

$\sin 45° = \frac{\sqrt{2}}{2}$, and B is acute, so B = 45°

Answer: 45°

6 Tree diagrams without replacement

A bag contains 5 orange beads and 3 purple beads.
Two beads are taken at random without replacement.
Work out the probability that (a) both are the same colour, (b) they are different colours.

(a) Add the ways: $\frac{5}{8} \times \frac{4}{7} + \frac{3}{8} \times \frac{2}{7} = \frac{13}{28}$

(b) P(different) = 1 − P(same) = $1 - \frac{13}{28} = \frac{15}{28}$

Answer: (a) $\frac{13}{28}$ (b) $\frac{15}{28}$

7 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 4 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 16 ÷ 4 = 4 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 4 \times 16 + 16 \times 10 + \frac{1}{2} \times 4 \times 16 = 32 + 160 + 32$
= 224

Answer: (a) 4 m/s^2 (b) 224 m

8 Reverse percentages

The price of a watch is increased by 40%.
Later the new price is reduced by 25% in a sale.
The sale price is £252.
Work out the price before the increase. (Calculator)

Multipliers: 1.4 and 0.75
Overall multiplier = 1.4 × 0.75 = 1.05
Original × 1.05 = £252
Original = £252 ÷ 1.05 = £240
Answer: £240

9 Factorising fully

Factorise fully $2x^2 - 10x + 8$

Take out the common factor 2: $2(x^2 - 5x + 4)$
Factorise the quadratic: $2(x - 1)(x - 4)$
Answer: $2(x - 1)(x - 4)$

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10 Rationalising the denominator

Rationalise the denominator of $\frac{26}{3\sqrt{6}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{6}$:

$\frac{26}{3\sqrt{6}} = \frac{26\sqrt{6}}{18}$

$= \frac{13\sqrt{6}}{9}$

Answer: $\mathbf{\frac{13\sqrt{6}}{9}}$