Day 254 **S Tier 10-a-day: Model Solutions**

1 Distance-time graphs

The graph shows Hassan's cycle ride to a café and back.
(a) Work out Hassan's speed on the way back.
(b) Work out Hassan's average speed for the whole ride, including the stop.

(a) Return: 12 km in 1.5 hours, so speed = 12 ÷ 1.5 = 8 km/h
(b) Total distance = 2 × 12 = 24 km, total time = 2.5 hours
Average speed = 24 ÷ 2.5 = 9.6 km/h
Answer: (a) 8 km/h (b) 9.6 km/h

2 Equations with fractions

Solve $\frac{x+7}{2} + \frac{x-6}{4} = 8$

Multiply every term by 4: $2(x + 7) + (x - 6) = 32$
Expand: $3x + 8 = 32$
$3x = 24$
$x = 8$
Answer: $x = 8$

3 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 40 cm. (Calculator)
The outer radius is 3 cm and the inner radius is 2.5 cm.
The density of the metal is 2.7 g/cm^3.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 3^2 \times 40 - \pi \times 2.5^2 \times 40 = \pi(9 - 6.25) \times 40$
= 345.58 cm^3
Mass = density × volume = 2.7 × 345.58 = 933.05 g
= 0.933 kg (3 s.f.)
Answer: 0.933 kg

4 Profit and loss

A retailer buys a bike and adds a 50% mark-up to get the selling price.
In a sale the selling price is reduced by 30% to £262.50.
Work out how much the shop paid for a bike.

Overall multiplier = 1.5 × 0.7 = 1.05
Cost = £262.50 ÷ 1.05
= £250
Answer: £250

5 Cumulative frequency graphs

The cumulative frequency graph gives information about the times of 80 students to complete a puzzle.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 80 had a time greater than 5.

(a) Median at cumulative frequency 40, so median = 30
Lower quartile at 20, so LQ = 20. Upper quartile at 60, so UQ = 40
Interquartile range = 40 − 20 = 20
(b) At 5, cumulative frequency = 7
Number greater = 80 − 7 = 73
Answer: (a) median 30, IQR 20 (b) 73

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6 Inverse functions

$f(x) = \frac{4x + 6}{x - 3}$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(2)$.

(a) Let $y = \frac{4x + 6}{x - 3}$. Then $y(x - 3) = 4x + 6$

$xy - 3y = 4x + 6$

Collect x terms: $xy - 4x = 6 + 3y$

Factorise: $x(y - 4) = 3y + 6$, so $x = \frac{3y + 6}{y - 4}$

$f^{-1}(x) = \frac{3x + 6}{x - 4}$

(b) $f^{-1}(2) = \frac{3 \times 2 + 6}{1 \times 2 - 4} = \frac{12}{-2} = -6$

Answer: (a) $f^{-1}(x) = \frac{3x + 6}{x - 4}$ (b) -6

7 Rates of work

Tap A can fill an empty tank in 20 minutes.
Tap B can fill an empty tank in 60 minutes.
Both are turned on at the same time.
How long do they take to fill an empty tank?

In 1 minute: Tap A does $\frac{1}{20}$ and Tap B does $\frac{1}{60}$

Together: $\frac{1}{20} + \frac{1}{60} = \frac{80}{1200}$

Time = $\frac{1200}{80}$ = 15 minutes

Answer: 15 minutes

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 25$.
The point P(−4, −3) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{3}{4}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{3}{4} = -\frac{4}{3}$

$y - (-3) = (-\frac{4}{3})(x - (-4))$

Multiply out and rearrange: $4x + 3y = -25$

(b) On the x-axis y = 0, so $4x = -25$

$x = -\frac{25}{4}$, so T is $(-\frac{25}{4}, 0)$

Answer: (a) $4x + 3y = -25$ (b) T = $(-\frac{25}{4}, 0)$

9 Surds with unknowns

Given that $(a + \sqrt{5})^2 = 14 + 6\sqrt{5}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{5})^2 = a^2 + 2a\sqrt{5} + 5$

Compare the $\sqrt{5}$ terms: $2a = 6$, so $a = 3$

Check the integer terms: $a^2 + 5 = 9 + 5 = 14$

Answer: a = 3

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10 Equal roots and real roots

The equation $2x^2 + 4x + k = 0$ has two different real solutions.
Find the range of possible values of k.

Two different real solutions means $b^2 - 4ac > 0$
$4^2 - 4 \times 2 \times k > 0$, so $16 - 8k > 0$
$16 > 8k$, so $k < 2$
Answer: $k < 2$