Day 253 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The sum of the interior angles of a polygon is 2520°.
Work out the number of sides of the polygon.

Sum of interior angles = (n − 2) × 180°
(n − 2) × 180 = 2520
n − 2 = 2520 ÷ 180 = 14
n = 16
Answer: 16

2 Capture and recapture

80 beetles in a field are caught, marked and put back.
Later, a second sample of 40 beetles is caught. 4 of these are marked.
(a) Work out an estimate for the number of beetles in a field.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 4 out of 40
This should match the proportion marked in the whole population: $\frac{4}{40} = \frac{80}{N}$
N = 80 × 40 ÷ 4 = 800
(b) No beetles are born, die or leave between the samples, and the beetles mix evenly
Answer: (a) 800 (b) The population stays the same between samples

3 Solving quadratics by factorising

Solve $3x^2 + 4x - 7 = 0$

Factorise: (3x + 7)(x − 1) = 0
3x + 7 = 0 or x − 1 = 0
$x = -\frac{7}{3}$ or x = 1
Answer: $x = -\frac{7}{3}$ or x = 1

4 Tests of divisibility

The 4-digit number 33k6 is a multiple of 3.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 3 if its digit sum is a multiple of 3
Digit sum = 3 + 3 + 6 + k = 12 + k
This must be a multiple of 3, so k = 0 or 3 or 6 or 9
Answer: k = 0 or 3 or 6 or 9

5 Exchange rates

Callum changes £300 into euros at £1 = €1.3.
Callum spends €180 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Callum get back?

£300 × 1.3 = €390
390 − 180 = €210
210 ÷ 1.6 = £131.25
Answer: £131.25

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6 Composite functions

$f(x) = 3x - 1$ and $g(x) = \frac{4}{x}$

(a) Find gf(x).
(b) Solve $gf(x) = -4$

(a) $gf(x) = g(3x - 1) = \frac{4}{3x - 1}$

(b) $\frac{4}{3x - 1} = -4$, so $3x - 1 = 4 \div -4 = -1$
$3x = 0$
$x = 0$

Answer: (a) $\frac{4}{3x - 1}$ (b) x = 0

7 Trigonometric graphs

The diagram shows the graph of $y = a \sin x + c$.
(a) Write down the value of a and the value of c.
(b) Work out the value of y when $x = 30°$.

(a) The maximum value is 7 and the minimum value is -3
$a = (7 - (-3)) \div 2 = 5$
$c = (7 + (-3)) \div 2 = 2$

(b) $y = 5 \sin 30° + 2$, and $\sin 30° = \frac{1}{2}$

$y = \frac{9}{2}$

Answer: (a) a = 5, c = 2 (b) y = $\frac{9}{2}$

8 Bounds of roots

The area of a circle is 238 cm^2, correct to the nearest cm^2.
Work out the lower bound of the radius of the circle.
Give your answer correct to 3 significant figures. (Calculator)

$237.5 \leq \text{area} < 238.5$

Area = πr^2, so $r = \sqrt{\frac{\text{area}}{\pi}}$

Lower bound = $\sqrt{\frac{237.5}{\pi}} = 8.69475... = 8.69$ (3 s.f.)

Answer: 8.69 cm

9 Exact trigonometric ratios from a given ratio

x is an acute angle and $\cos x = \frac{5}{8}$.

(a) Find the exact value of sin x.
(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\sin^2 x + \cos^2 x = 1$, so $\sin^2 x = 1 - (\frac{5}{8})^2 = \frac{39}{64}$

x is acute, so sin x is positive: $\sin x = \frac{\sqrt{39}}{8}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{39}}{8} \div \frac{5}{8} = \frac{\sqrt{39}}{5}$

Answer: (a) $\frac{\sqrt{39}}{8}$ (b) $\frac{\sqrt{39}}{5}$

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10 Nested surds

Show that $\sqrt{24 + 16\sqrt{2}} = 4 + 2\sqrt{2}$

Square the right-hand side: $(4 + 2\sqrt{2})^2 = 16 + 16\sqrt{2} + 8$

$= 24 + 16\sqrt{2}$

So $\sqrt{24 + 16\sqrt{2}} = 4 + 2\sqrt{2}$

Answer: $\sqrt{\mathbf{24 + 16\sqrt{2}}} = \mathbf{4 + 2\sqrt{2}}$