Day 252 **S Tier 10-a-day: Model Solutions**

1 Changing the subject

Make r the subject of $P = \frac{9}{r^2}$

where r is positive.

Multiply by r^2: $Pr^2 = 9$

Divide by P: $r^2 = \frac{9}{P}$

Square root: $r = \sqrt{\frac{9}{P}}$

Answer: $r = \sqrt{\frac{9}{P}}$

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (4x + 10)°
Angle PQR = (x + 5)°
Angle PRS = (4x + 40)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
4x + 40 = 5x + 15
x = 25
(b) Angle PRS = 140° so angle PRQ = 180° − 140° = 40°
Answer: (a) x = 25 (b) 40°

3 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 25 is taken. Work out how many come from Year 7.
(b) A different stratified sample has 9 from Year 8. Work out the size of this sample.

Total = 450 + 150 + 150 + 500 = 1250

(a) Year 7: $\frac{450}{1250} \times 25 = 9$

(b) Fraction sampled = 9 ÷ 150 = $\frac{3}{50}$, so sample size = $\frac{3}{50} \times 1250 = 75$

Answer: (a) 9 (b) 75

4 Solving quadratics by factorising

Solve $4x^2 - 5 = -19x$

Rearrange so that one side is 0: $4x^2 + 19x - 5 = 0$
Factorise: $(x + 5)(4x - 1) = 0$
$x + 5 = 0$ or $4x - 1 = 0$

$x = -5$ or $x = \frac{1}{4}$

Answer: $x = -5$ or $x = \frac{1}{4}$

5 Percentages

R is 30% less than S.
S is 15% less than A.
Work out R as a percentage of A.

R = 0.7 × S and S = 0.85 × A
R = 0.7 × 0.85 × A = 0.595 × A
0.595 × 100 = 59.5%
Answer: 59.5%

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6 Similar shapes

Two cones, P and Q, are mathematically similar.
The volume of P is 40 cm^3 and the volume of Q is 135 cm^3.
The curved surface area of P is 32 cm^2.
Work out the curved surface area of Q.

Ratio of volumes = 40 : 135 = $2^3 : 3^3$
So the ratio of lengths is 2 : 3

Ratio of areas = $2^2 : 3^2$, so scale factor = $\frac{9}{4}$

$32 \times \frac{9}{4} = 72$ cm^2

Answer: 72 cm^2

7 Iteration: rearranging equations

(a) Show that the equation $x^2 - x - 8 = 0$ has a solution between x = 3 and x = 4.
(b) Show that the equation can be rearranged to give $x = 1 + \frac{8}{x}$

(a) Let $f(x) = x^2 - x - 8$. $f(3) = 3^2 - 3 - 8 = -2$ and $f(4) = 4^2 - 4 - 8 = 4$
f(3) is negative and f(4) is positive. There is a change of sign, so there is a solution between 3 and 4
(b) Rearrange $x^2 - x - 8 = 0$: $x^2 - x = 8$
Factorise: $x(x - 1) = 8$
Divide by x: $x - 1 = \frac{8}{x}$, so $x = 1 + \frac{8}{x}$

Answer: Shown

8 Laws of indices

Given that $\frac{5^3 \times 25^2}{25^3} = 5^n$,
find the value of n.

$25 = 5^2$
Numerator: $5^3 \times 5^4 = 5^7$
Denominator: 5^6
Subtract the powers: 7 − 6 = 1
Answer: n = 1

9 Combined transformations

The point P(−3, −6) is rotated 90° clockwise about the origin.
The image is then reflected in the line y = x.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−6, 3)
After the second transformation: P'' = (3, −6)
(b) Follow a general point (x, y) through both transformations: (x, y) → (y, −x) → (−x, y)
The single mapping is (x, y) → (−x, y)
This is a reflection in the y-axis
Answer: (a) (3, −6) (b) Reflection in the y-axis

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10 Rates of work

Pipe A can fill an empty pool in 24 minutes.
Pipe B can fill an empty pool in 40 minutes.
Both are turned on at the same time.
How long do they take to fill an empty pool?

In 1 minute: Pipe A does $\frac{1}{24}$ and Pipe B does $\frac{1}{40}$

Together: $\frac{1}{24} + \frac{1}{40} = \frac{64}{960}$

Time = $\frac{960}{64}$ = 15 minutes

Answer: 15 minutes