Day 251 **S Tier 10-a-day: Model Solutions**

1 Fractions

Work out $(5\frac{1}{2} - 1\frac{1}{4}) \times \frac{2}{3}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{11}{2} - \frac{5}{4} = \frac{17}{4}$

$\frac{17}{4} \times \frac{2}{3} = \frac{17}{6}$

$= 2\frac{5}{6}$

Answer: $2\frac{5}{6}$

2 Substitution

Work out the value of $\frac{2m - n}{k}$

when $m = 1$, $n = -1$ and $k = -4$.

Substitute the values: $\frac{2 \times 1 - (-1)}{-4}$

$= \frac{3}{-4}$

$= -\frac{3}{4}$

Answer: $-\frac{3}{4}$

3 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 80 cm. (Calculator)
The outer radius is 8 cm and the inner radius is 6.5 cm.
The density of the metal is 8.5 g/cm^3.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 8^2 \times 80 - \pi \times 6.5^2 \times 80 = \pi(64 - 42.25) \times 80$
$= 5466.4$ cm^3
Mass = density × volume = $8.5 \times 5466.4 = 46464$ g
= 46.5 kg (3 s.f.)
Answer: 46.5 kg

4 Inverse functions

$f(x) = \frac{3}{x - 4} + 1$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(0)$.

(a) Let $y = \frac{3}{x - 4} + 1$, so $y - 1 = \frac{3}{x - 4}$

Make x the subject: $x - 4 = \frac{3}{y - 1}$

$x = \frac{3}{y - 1} + 4$

$f^{-1}(x) = \frac{3}{x - 1} + 4$

(b) $f^{-1}(0) = \frac{3}{0 - 1} + 4 = 1$

Answer: (a) $f^{-1}(x) = \frac{3}{x - 1} + 4$ (b) 1

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5 Using a line of best fit

The scatter graph shows hours of revision (x) and test mark (y).
The equation of the line of best fit is y = 3x + 12.
(a) Estimate y when x = 9.
(b) Explain why an estimate of y when x = 25 may be unreliable.

(a) $y = 3 \times 9 + 12 = 39$
(b) x = 25 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 39 (b) x = 25 is outside the range of the data

6 Exact trigonometric values

Work out the exact value of $(\cos 60°)^2 + \tan 60°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$(\cos 60°)^2 + \tan 60° = (\frac{1}{2})^2 + \sqrt{3}$

$= \frac{1}{4} + \sqrt{3}$

Answer: $\mathbf{\frac{1}{4} + \sqrt{3}}$

7 Fractions of fractions

Gareth cycles $\frac{2}{3}$ of a route before lunch.

After lunch he cycles $\frac{3}{5}$ of the remaining distance.

He still has 22 km to go.
How long is the route, in km?

After the first part, $\frac{1}{3}$ remains

After the second, $\frac{2}{5}$ of $\frac{1}{3}$ remains = $\frac{2}{15}$ of the total

$\frac{2}{15}$ of the total is 22

Total = $22 \div \frac{2}{15} = 165$

Answer: 165 km

8 Conditional probability from a table

The table shows information about 77 employees.
(a) An employee who walks to work is chosen at random. Work out the probability that this employee is a woman.
(b) Two of the 77 employees are chosen at random. Work out the probability that both are women.

(a) 31 employees are in the Walk column, and 9 of them are women

Probability = $\frac{9}{31}$

(b) There are 38 women, so P(both) = $\frac{38}{77} \times \frac{37}{76} = \frac{37}{154}$

Answer: (a) $\mathbf{\frac{9}{31}}$ **(b)** $\mathbf{\frac{37}{154}}$

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9 Bounds of roots

The area of a circle is 342 cm^2, correct to the nearest cm^2.
Work out the lower bound of the radius of the circle.
Give your answer correct to 3 significant figures. (Calculator)

$341.5 \leq \text{area} < 342.5$

Area = πr^2, so r = $\sqrt{\frac{\text{area}}{\pi}}$

Lower bound = $\sqrt{\frac{341.5}{\pi}}$ = 10.4261... = 10.4 (3 s.f.)

Answer: 10.4 cm

10 Circle and line intersections

Solve the simultaneous equations

$x^2 + y^2 = 50$

$y = -x + 8$

Substitute into the first equation: $x^2 + (-x + 8)^2 = 50$

Expand: $2x^2 - 16x + 14 = 0$

Divide by 2: $x^2 - 8x + 7 = 0$

Factorise: $(x - 1)(x - 7) = 0$, so x = 1 or x = 7

$y = -x + 8$ gives y = 7 and y = 1

Answer: x = 1, y = 7 and x = 7, y = 1