Day 250 **S Tier 10-a-day: Model Solutions**

1 Distance-time graphs

The graph shows Leila's cycle ride to a café and back.
(a) Work out Leila's speed on the way back.
(b) Work out Leila's average speed for the whole ride, including the stop.

(a) Return: 24 km in 3 hours, so speed = 24 ÷ 3 = 8 km/h
(b) Total distance = 2 × 24 = 48 km, total time = 5 hours
Average speed = 48 ÷ 5 = 9.6 km/h
Answer: (a) 8 km/h (b) 9.6 km/h

2 HCF and LCM

The HCF of two numbers is 4 and their LCM is 168.
One of the numbers is 24.
Work out the other number.

HCF × LCM = product of the two numbers
4 × 168 = 672
Other number = 672 ÷ 24 = 28
Answer: 28

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 23°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 23°
Angle AOB = 180 − 2 × 23 = 134°
(b) The angle at the circumference is half the angle at the centre: 134 ÷ 2 = 67°
Answer: (a) 134° (b) 67°

4 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 2 : 3.
50% of the Year 10 students and 45% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 100 students in total: 40 Year 10 students and 60 Year 11 students
50% of 40 = 20 and 45% of 60 = 27
Together: 47 out of 100
47 ÷ 100 × 100 = 47%
Answer: 47%

5 Perpendicular bisector

A is the point (4, −1) and B is the point (−2, 3).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (1, 1)

Gradient of AB = $-\frac{2}{3}$

Perpendicular gradient = $\frac{3}{2}$

$y - 1 = \frac{3}{2}(x - 1)$

Rearrange to integer coefficients: 3x − 2y = 1
Answer: 3x − 2y = 1

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6 Laws of indices

Write $\frac{8^{n-1} \times 2^{n+3}}{2^n}$ in the form 2^{an+c}

where a and c are integers.

$8 = 2^3$, $2 = 2^1$, $2 = 2^1$

$8^{n-1} = 2^{3(n-1)}$, $2^{n+3} = 2^{n+3}$, $2^n = 2^n$

Exponent: $3(n-1) + (n+3) - n = 3n$

Answer: a = 3, c = 0

7 Composite functions: fg and gf

$f(x) = -3x - 5$ and $g(x) = x^2 - 3$

Solve $fg(x) = gf(x)$

$fg(x) = f(x^2 - 3) = -3x^2 + 4$

$gf(x) = g(-3x - 5) = (-3x - 5)^2 - 3 = 9x^2 + 30x + 22$

Set them equal and rearrange: $12x^2 + 30x + 18 = 0$

Divide by 6 and factorise: $(x + 1)(2x + 3) = 0$

$x = -\frac{3}{2}$ or $x = -1$

Answer: $x = -\frac{3}{2}$ or $x = -1$

8 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)

(a) Work out the length OM.

(b) Work out angle AOB, to 1 decimal place.

The diagram is not drawn accurately.

(a) AM = 30 ÷ 2 = 15 cm

$OM^2 = OA^2 - AM^2 = 17^2 - 15^2 = 64$

OM = 8 cm

(b) $\sin(\text{angle AOM}) = \frac{15}{17}$

Angle AOM = 61.93°, so angle AOB = 2 × 61.93 = 123.9°

Answer: (a) 8 cm (b) 123.9°

9 Compound interest

Lena puts £4,000 in a savings account.

Compound interest is paid at 3% in year 1, 3% in year 2 and 2% in year 3.

Work out the amount in the account at the end of year 3.

Give your answer to the nearest penny. (Calculator)

Multipliers: 1.03, 1.03, 1.02

£4,000 × 1.03 × 1.03 × 1.02

= £4,328.47

Answer: £4,328.47

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10 Direct and inverse proportion

T is directly proportional to the square of l.
When l = 2, T = 36.
(a) Find a formula for T in terms of l.
(b) Find the value of l when T = 225.

(a) $T = kl^2$

Substitute l = 2 and T = 36: $36 = k \times 2^2$

$k = 9$

$T = 9l^2$

(b) $225 = 9l^2$, so $l^2 = 225 \div 9 = 25$, so $l = 5$

Answer: (a) $T = 9l^2$ (b) $l = 5$